IMPROVING NUCLEAR POWER PLANT SAFETY AND OPERATING MARGINS THROUGH LINEAR QUADRATIC ESTIMATION

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ABSTRACT

Redundant measurement, trip, and actuation channels are required for reactor trips and engineered safety systems in nuclear power plants. There are three or four independent channels in existing plants, depending on the system. To prevent failures, consistent signals from any two channels together can trip the reactor or actuate safety systems. Measurements associated with safety trips and actuations include uncertainties related to the instruments making the measurements and to process conditions. Margins that include measurement uncertainties are set for each trip and actuation channel as though it were the only one taking action, thus preserving independence. There are sensors, however, that provide partially redundant information that could be used to decrease the uncertainty associated with measurements of plant conditions, while maintaining the three-channel redundancy important to plant safety. Reducing these uncertainties would both improve plant safety by providing more accurate knowledge of plant conditions and improve the operating margins that must take such uncertainties into account. This paper demonstrates how information that is already available to plant instrumentation and control systems and is already redundant can be used through software routines to reduce the error that must be taken into account in reactor trip and safety system actuation setpoints. Issues that would have to be addressed related to standards, licensing, and implementation are also discussed.

Key Words: Nuclear Safety System, Operating Margins, Linear Quadratic Estimation, Kalman Filtering, Measurement Uncertainty

1 INTRODUCTION

All measurements include some uncertainty, and measurement uncertainties are an important part of determining reactor trip and engineered safeguards setpoints in nuclear power plants. Although signals and trip and actuation channels are redundant, allowances for measurement uncertainties associated with each channel are included as though there were no redundancy in measurements, to ensure that sufficient margin is included in safety actions even with instrument failures. In existing, operating plants, there are three or four channels of measurements, usually four. Even within each channel's set of measurements, however, there are measurements that could be combined through the proper mathematical and engineering techniques to use redundant information to make estimates of plant conditions that are better than any individual direct measurement.

The Linear Quadratic Estimation (LQE) technique (also called the Kalman filter) was pioneered in the early 1960s and could be used to combine measurements that are already redundantly channelized but that measure different process states to develop estimates of safety-critical plant states. LQE saw its first application in the aerospace industry, particularly spacecraft and missile guidance. The pioneer in its development and application to control systems was R. E. Kalman. Reference [2] is the original paper published by Kalman in 1960. Reference [3] was published shortly thereafter, refining and optimizing the technique further. LQE uses previous data about the process's performance along with current data related to process inputs to estimate the subject variable. The routine then updates the model used to determine the estimate based on the current direct measurement of the estimated variable. The updated model is used in the next estimate, and the process of estimating, measuring, and updating is continued. A priori knowledge about the process system's characteristics can lead to more rapid convergence of the model but is not required for LQE to operate correctly.

LQE based on multiple process variables would be more accurate (include less measurement uncertainty) than the actual measurements of the plant states in a nuclear power plant from single instruments. For example, steam generator (SG) level could be estimated both by measuring the level directly and by measuring the feedwater (FW) flow into and steam flow out of each SG.

One condition that trips the reactor in a pressurized water reactor (PWR) plant is low SG level as measured by narrow range SG level sensors. SG level's behavior in U-tube SGs under various plant conditions is very well-understood and very well-documented in the technical literature. The authors of this paper have themselves studied U-tube SG level behavior in academic settings and done research into ways to reduce PWR plant trips caused by SG level excursions. For that reason, SG level has been chosen to illustrate the application of LQE in reducing the uncertainty associated with plant measurements. SG level is measured directly in all PWR plants. It depends primarily on the mismatch between the mass flows of FW into the SG and steam out of it. Its behavior is also influenced by other conditions that are related to reactor power level such as steam pressure, reactor coolant flow, and reactor coolant temperature. For various power levels, detailed, robust models have been developed for U-tube SG level behavior based on FW and steam mass flows, under both steady state and transient conditions. Using measurements of FW flow and steam flow combined with SG level measurement itself, an LQE will be developed in this paper. It will be shown that the LQE tracks actual SG level much more closely than direct measurements of SG level that include uncertainty. In this example, it is assumed that three direct measurements of SG level are made, recognizing that four rather than three measurements are often made. The results of the LQE estimate are not dependent on the number of direct measurements of the state of concern that are made. It is important to note in this analysis that what is being proposed is not cross-channel communication between transmitters measuring the same variable, but measurements of different process states made by instruments within the same channel.

FW flow is measured in every PWR plant, but it is not always a redundant, nuclear-safety-related (SR) measurement. Steam flow likewise is sometimes not measured with redundant, SR instruments. In order to be used in an LQE algorithm in triggering an SR action, both measurements would have to be present, redundant, and made by SR instruments. There are many SR measurements that have related measurements made by other SR instruments, and those process variables could benefit by the application of LQE with related SR measurements. U-tube SG level behavior is chosen in this paper to demonstrate the advantages of LQE because its behavior is thoroughly modeled and documented in the literature; and because modeling it requires a relatively high-ordered model, requiring a large number of calculations relative to other potential SR setpoints that could use LQE.

2 ASSUMPTIONS

The following simplifying assumptions are made in this paper.

1. The parameters used in state equations representing U-tube SG level behavior are heavily dependent on overall plant power level, being affected in non-linear ways by changes in power levels. For this demonstration, it assumed that the plant is running at 100% power. If a system were implemented over the full power range, changes to the model determined by

power level would have to be incorporated into the total software system. Methods that implement adjustable parameters like those that would be required to encompass a plant's full power range have been developed, and literature citations on such methods can be provided.

2. Uncertainties related to measurements in SR applications are normally stated as a percentage of the measurement span. (e.g., for a pressure measurement ranging from 50 to 200 PSIG, error would be stated as percent of the 150 PSIG span.) Uncertainties are broken into two parts, one expressed as a bias related to process or ambient conditions, and one as a random, normally distributed, unbiased uncertainty. The random part is expressed as the 95/95 confidence interval of the manufacturer's calibration and test data, roughly stated as the interval encompassing + two standard deviations of the data obtained from performance tests. LQE literature and algorithms refer to the measurement covariance, and the measurement covariance can roughly be said to be the square root of half of the stated measurement uncertainty.

In this paper it is assumed that the uncertainty is completely random, normal, and unbiased. Bias will need to be addressed before bringing the concept of LQE into implementation. Methods that make LQE robust in the presence of biases in measurements have been developed and published.

3 ACRONYMS AND DEFINITIONS

Table I. Acronyms

Acronym	Definition
COTS	Commercial Off-the-Shelf
FW	Feedwater
LQE	Linear Quadratic Estimation
MIMO	Multi-Input Multi-Output
NRC	Nuclear Regulatory Commission
SR	Nuclear-Safety-Related
PWR	Pressurized Water Reactor
RCS	Reactor Coolant System
SG	Steam Generator
SISO	Single-Input Single-Output

4 METHOD

The following are the steps taken in this paper:

1. A model of U-tube SG level behavior that has been documented in the technical literature and subjected to further testing and validation is explained. That model relates SG level to steam mass flow and FW mass flow under different plant conditions. The model parameters developed for 100% plant power are used here, and an analog state space representation of the system is developed from the original authors' transfer function model. A discrete state space model is then developed using a sampling time of 0.2 seconds.

2. In a PWR plant, steam flow is actually determined by a number of factors of which FW flow is only one. To demonstrate LQE in this paper, steam flow is modeled to follow FW flow with a one second exponential lag.

3. A transient in FW flow is simulated. The transient consists of four 2% step-pulse increases in FW flow, each step 2 seconds in duration, and separated from the previous one by six seconds. The process response is simulated for a total of twenty (20) seconds from the time the first FW transient was introduced.
4. Small and continuous variations in process conditions are simulated by adding random noise to the FW and steam mass flows. The noise is random, unbiased, and normally distributed, with a standard deviation of 0.05%. In the simulation, process noise in FW flow is carried forward also to influence steam flow, and the process noise associated with steam flow directly is then added.
5. The simulation results are plotted, showing FW mass flow, steam mass flow, and SG level.
6. Direct measurements of SG level, FW flow, and steam flow that include unbiased, normally distributed, random noise are simulated, which would mimic actual measurements taken by SR instruments. The measurement noise's standard deviation is assumed as 0.125%, roughly equivalent to +0.25% error stated in terms of a 95/95 confidence interval. The measurements are then combined to show that the combined (averaged) behavior of three independent measurements is consistent with what would be expected based on statistical and metrological techniques.
7. Measurements of FW and steam mass flow that include noise from measurement uncertainty are then combined with a direct, noisy SG level measurement in an LQE algorithm to show that the LQE includes significantly less uncertainty than a single, direct, noisy SG level measurement. The model output of SG level, one of the noisy SG level measurements, and the LQE of SG level are then plotted on the same graph.

5 SG LEVEL MODEL

In 1980, Irving and Bihoreaux [1] developed a set of sixth-order models of U-tube SG level behavior, each member of the set taking differing process behavior at different plant power levels into account. The models were evaluated and used by, among others, Kim et al. in their 1993 paper [4]. Other citations of Irving's and Bihoreaux's original paper are in the literature and can be provided on request. Irving and Bihoreaux described U-tube SG level behavior in the s-domain by (1).

$$L(s) = \frac{G_1}{s}\left(Q_e(s) - Q_v(s)\right) - \frac{G_2}{1+\tau_2 s}\left(Q_e(s) - KQ_v(s)\right) + \frac{G_3 s}{\tau^{-2} + 4\pi^2 T^{-2} + 2\tau^{-1}s + s^2} Q_e(s) \tag{1}$$

In this equation L is normalized SG level, Q_e is normalized FW mass flow into the SG, and QV is normalized steam mass flow out.

For 100% steam flow (100% power), the equation parameters for the modeled plant are as follows:

G_1=0.058, G_2=0.47, G_3=0.105, K=4.82, T=28.6, τ=11.7, τ_2=3.4

Re-written to isolate the steam flow and feed flow variables, the equation becomes (2):

$$L(s) = \left(\frac{G_1}{s} - \frac{G_2}{1+\tau_2 s} + \frac{G_3 s}{\tau^{-2} + 4\pi^2 T^{-2} + 2\tau^{-1}s + s^2}\right) Q_e(s) + \left(\frac{-G_1}{s} + \frac{KG_2}{1+\tau_2 s}\right) Q_v(s) \tag{2}$$

When the numeric values are substituted, the equation becomes (3):

$$L(s) = \left(\frac{0.058}{s} - \frac{0.47}{1+3.4s} + \frac{0.105s}{0.0556 + 0.171s + s^2}\right) Q_e(s) + \left(-\frac{0.058}{s} + \frac{2.27}{1+3.4s}\right) Q_v(s) \tag{3}$$

Changing (3) from a transfer function to a system of state equations, using discrete variables and a 0.2 second sampling time, results in the following system definition, where $\bar{x}(k)$ represents the discrete system states (vector), $\bar{u}(k)$ represents the discrete system inputs or controls (vector), and $y(k)$ represents the discrete system output. This presentation bypasses the transformation in the s-domain from transfer function to state equation, which was done via standard transformation techniques by manipulating (3) from a continuous-time transfer function to a continuous-time state space representation, then by converting the continuous-time state space model to a discrete-time state space model, as shown in (4). The transformation was made using Matlab 2014b.

$$\begin{gathered} \bar{x}(k+1) = A\bar{x}(k) + B\bar{u}(k) \\ y(k) = C\bar{x}(k) + D\bar{u}(k) \end{gathered} \tag{4}$$

where

$$\bar{x}(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \\ x_5(k) \\ x_6(k) \end{bmatrix}$$
$$\bar{u}(k) = \begin{bmatrix} q_e(k) \\ q_v(k) \end{bmatrix}$$

and

$$A = \begin{bmatrix} 0.909 & -0.0411 & -0.0125 & 0 & 0 & 0 \\ 0.0954 & 0.998 & -0.000634 & 0 & 0 & 0 \\ 0.00485 & 0.0999 & 1 & 0 & 0 & 0 \\ 0.000326 & 0.01 & 0.2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.943 & 0 \\ 0 & 0 & 0 & 0 & 0.194 & 1 \end{bmatrix}$$
$$B = \begin{bmatrix} 0.0477 & 0 \\ 0.00242 & 0 \\ 0.0000814 & 0 \\ 0.00000409 & 0 \\ 0 & 0.194 \\ 0 & 0.0196 \end{bmatrix}$$
$$C = [0.0991 \quad 0.274 \quad -0.0247 \quad 0.0152 \quad 0.61 \quad -0.0171]$$
$$D = 0.$$

As stated above, to allow for simulation, steam mass flow (q_v) was taken to equal FW mass flow (q_e) with a 1-second exponential lag. In a PWR plant operating in steady state at 100% power, this assumption would not be exactly true, but would be a reasonable estimation of steam flow operation. The discrete-time equation for the 1 second lag at 0.2 second sampling intervals is shown in (5):

$$q_v(k+1) = 0.8187q_v(k) + 0.1813q_e(k+1) \tag{5}$$

FW flow was perturbed by adding four step-pulse increases of 2%, each lasting two seconds, with six seconds between each of the perturbations. Process noise was simulated first by adding random, unbiased, normally distributed noise to the FW flow, carrying the noisy FW flow value through the steam flow simulation, and then adding random, unbiased, normally distributed noise to the steam flow. Process noise standard deviation was 0.05%. Using the perturbed, noisy inputs, SG level was simulated. The results are shown in Figure 1.

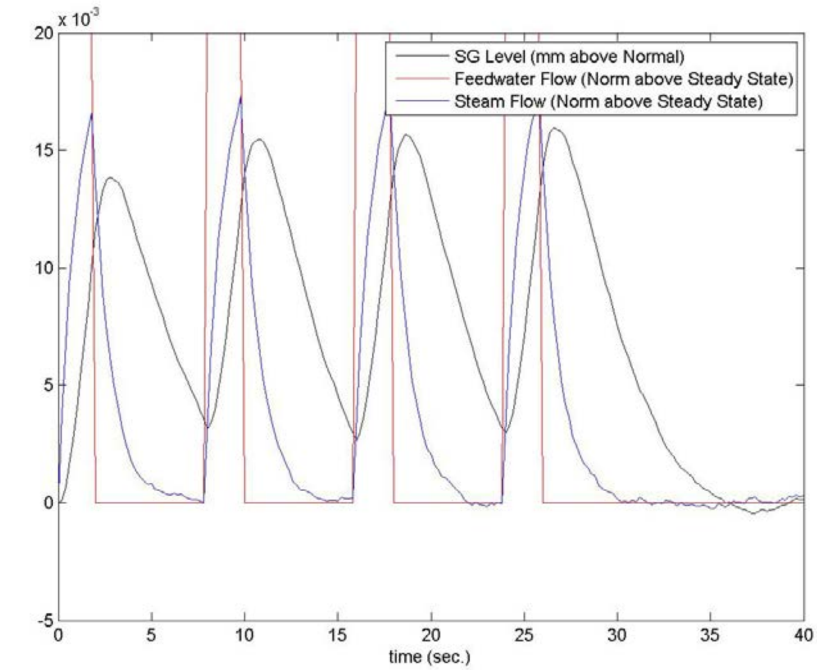

Figure 1. FW Flow, Steam Flow, and SG Level

6 SG LEVEL MODEL WITH INSTRUMENT UNCERTAINTIES INCLUDED

Three measurements of each model input and output were simulated by adding random, unbiased, normally distributed noise to the FW flow, steam flow, and SG level values. For each model input and output, the three noisy measurements were averaged. Figure 2, Figure 3, and Figure 4 show the noisy measurements and the averaged value for FW flow, steam flow, and SG level, respectively.

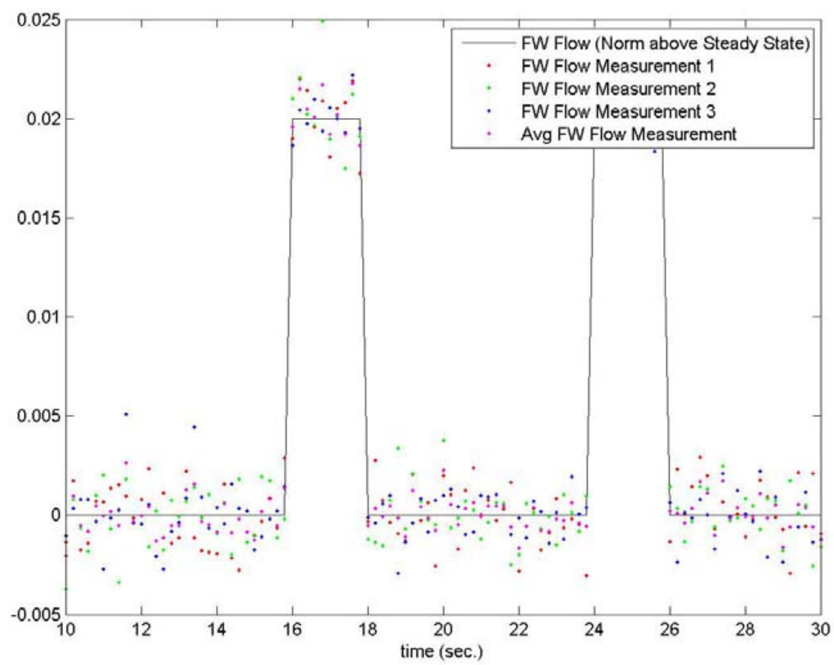

Figure 2. FW Flow with Noisy Measurements

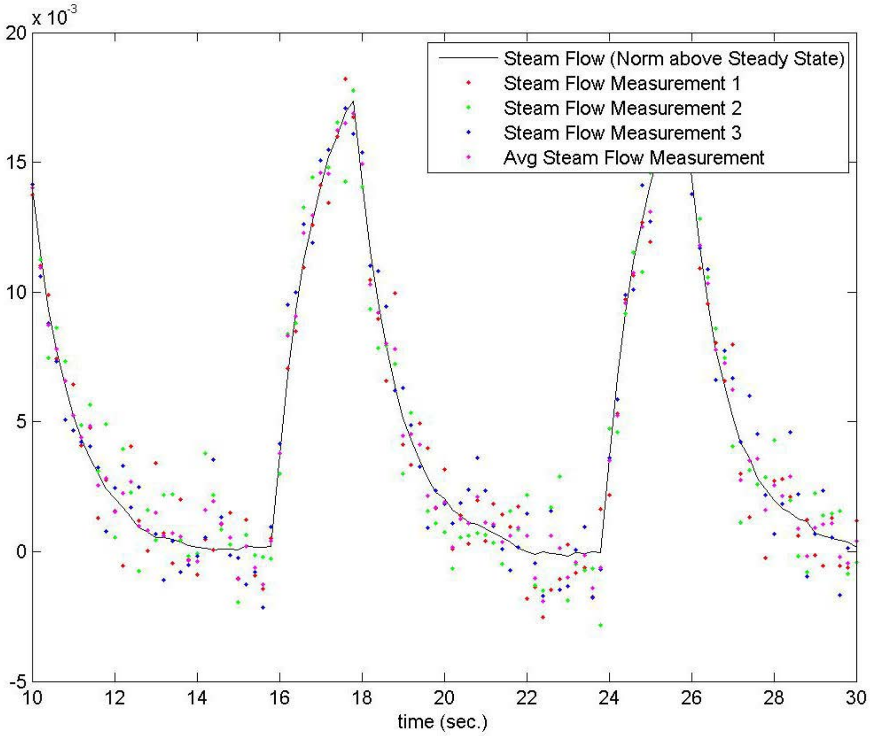

Figure 3. Steam Flow with Noisy Measurements

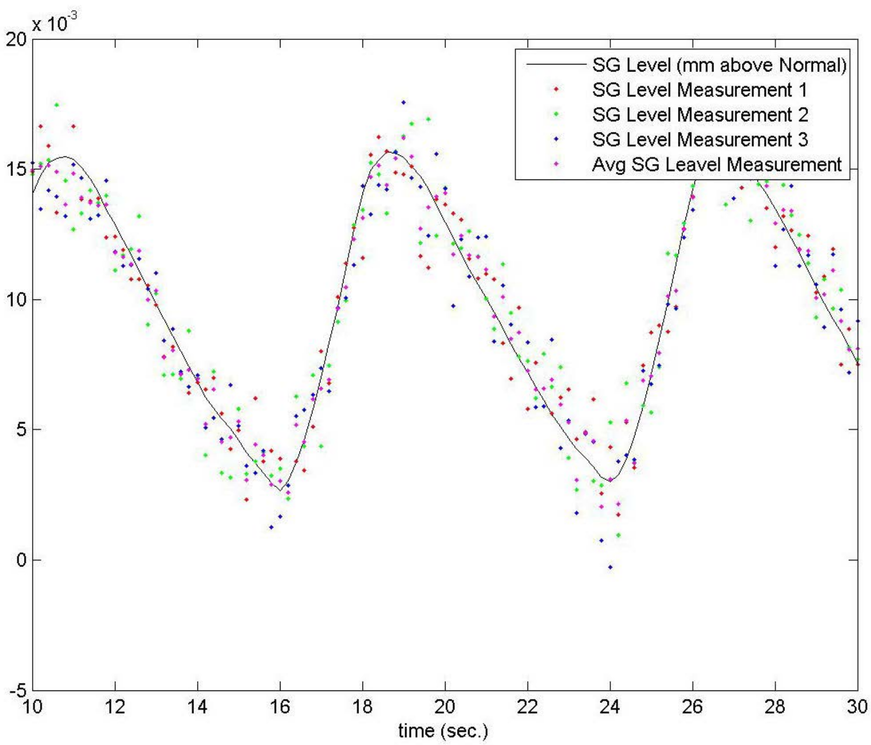

Figure 4. SG Level with Noisy Measurements

After generating the random sequences for each process variable, the bias and standard deviation of the sequences were calculated. Then the bias and standard deviation of the average sequence for each variable were calculated. Results are given in Table II. As expected, for each of the process variables the standard deviation of the average of the three measurements is smaller than the standard deviation of the individual measurement noises by a factor of roughly $\sqrt{3}$.

Table II. Statistical Characteristics of Simulated Measurement Uncertainty

Description	Bias of Noise Mean	Standard Deviation
Feedwater Flow		
Sequence 1	-8.1×10^{-5}	0.0014
Sequence 2	-6.1×10^{-6}	0.0014
Sequence 3	-2.8×10^{-6}	0.0013
Average Sequence	-3.0×10^{-5}	0.00089
Steam Flow		
Sequence 1	-6.4×10^{-5}	0.0013
Sequence 2	1.8×10^{-5}	0.0013
Sequence 3	-7.7×10^{-5}	0.0012
Average Sequence	-1.0×10^{-5}	0.00078
Steam Generator Level		
Sequence 1	-5.2×10^{-5}	0.0011
Sequence 2	-4.6×10^{-5}	0.0013
Sequence 3	-1.5×10^{-5}	0.0013
Average Sequence	-8.4×10^{-5}	0.00074

7 SG LEVEL MODEL WITH LQE OF LEVEL

There have been many step-by-step descriptions written on how to implement a Kalman filter in a computer program. The authors found that which was provided by Simon [5] to be one of the clearest and most succinct. Following Simon's method, a Kalman filter was developed for estimating SG level using noisy measurements of FW flow, steam flow, and SG level and implemented in Matlab 2014b. The original code is not included in this paper, but will be provided by the authors upon request. Figure 5 is a plot of the actual simulated SG level measurement, one of the noisy measurements of SG level, and the Kalman estimate.

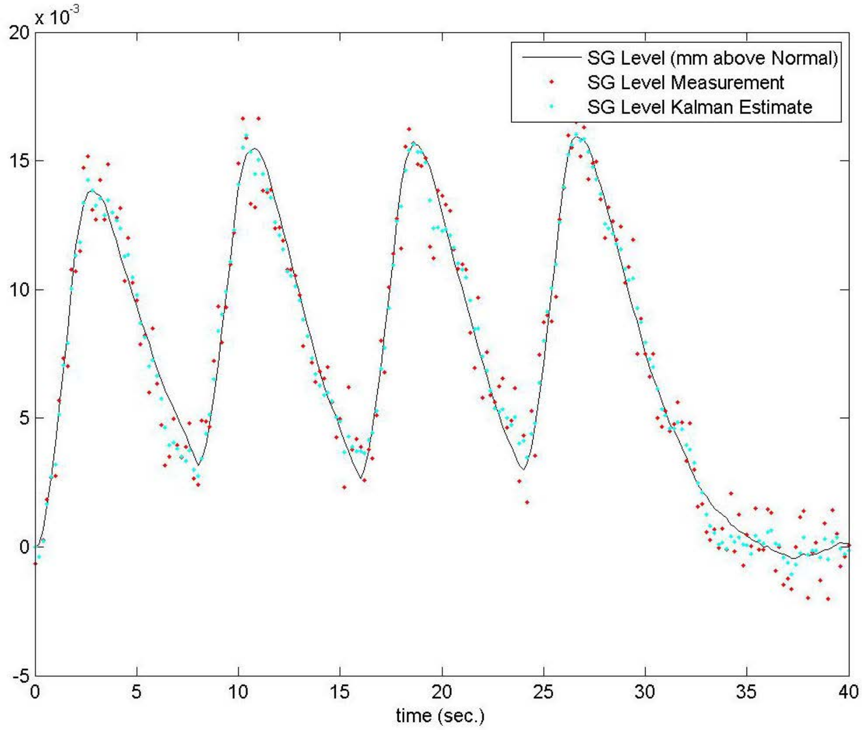

Figure 5. SG Level with Kalman Estimate

Table III lists the bias and standard deviations of the noisy SG level measurement used in the implementation. Note that the bias is essentially the same, and the standard deviation of the Kalman estimate is slightly more than half the standard deviation of the noisy measurement. The Kalman filter's estimate includes less uncertainty than the average of the three direct, redundant SG level measurements.

Table III. Comparison of Uncertainty of Individual Measurements and Kalman Filter Estimate

Description	Bias of Noise Mean	Standard Deviation
Steam Generator Level		
Sequence 1	-5.2×10^{-5}	0.0011
Kalman Filter Estimate (LQE)	-5.4×10^{-5}	6.0×10^{-4}

The best measurement of a Kalman filter's stability is the stability of the K matrix. The K matrix contains the correction factors applied to each of the states in the process model used in the filter. Ideally, each value in the K matrix will settle to a stable value or oscillate within a relatively small range. In this case, the model is sixth order, there are two inputs, and one estimate, so the K matrix is 8 x 1. Figure 6 is a plot of each value of the K matrix. It can be seen in this example that K is stable.

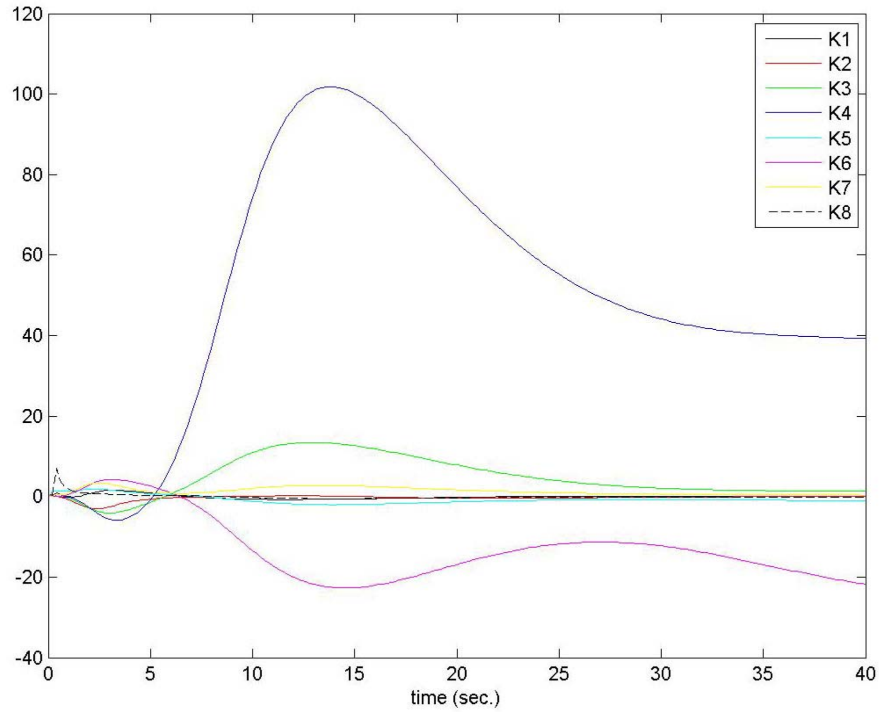

Figure 6. Kalman Gain Matrix

8 IMPLEMENTATION AND USE IN NUCLEAR POWER PLANTS

LQE is based on advanced statistical and mathematical techniques, but is computationally simple and requires relatively small amounts of computer memory, mainly in the form of square matrices. It is implemented by a recursive routine. In the Matlab implementation used in this paper, the entire estimator/Kalman filter is implemented in six lines of recursive code. For a system with n inputs, m outputs, and a model of order O, the routine in (6) requires intermediate variables.

$$4(O+m)^2 + 2(O+m)(n+1) + n^2 + 2n \tag{6}$$

Thus for a two-input, single-output SG level implementation based on a sixth order process model, 291 (i.e., 256+32+1+2) words of computer memory would be required. The operation of the Kalman filter described here would require roughly 2,500 floating point multiplications and a corresponding number of additions per recursion. Although process controllers and digital safety systems are not optimized for matrix operations, 2,500 discrete multiplications and 2,500 discrete additions do not together make a formidable task for such system processors. Furthermore, there are functions available that would streamline operation of the necessary code, and it is not necessary to update the model with every cycle of measurement. The Kalman filter code thus could be incorporated into blocks that would not delay measurement and trip functions beyond acceptable limits. As stated in the introduction, U-tube SG level behavior was chosen because it is a relatively high order system, requiring more calculations than lower order systems, yet it still results in manageable computational requirements.

The example given in this paper showed a reduction of measurement uncertainty by half through the use of LQE. The improvement seen in a given plant application would depend on the number of process variables influencing the estimate that are measured and the uncertainties of the measurements of those variables. Although each application would see an improvement by a different factor, the minimal improvement for each application can be calculated rigorously before implementation, given worst-case uncertainty allowances for each measurement affecting the estimate.

Although the technical implementation of LQE in nuclear power plants would be simple, there would be significant hurdles in adapting approved standards and current licensing expectations to its use in reducing measurement uncertainty. Nuclear Regulatory Commission (NRC) Regulatory Guide 1.105 Rev. 3 (1999) [6] and the corresponding ISA standard, ISA-S67.04-1994 [7] are based solely on the use of single measurements for each variable used in a safety trip or actuation channel with no data processing. Industry standards and regulatory guidance would have to be changed to incorporate LQE into SR measurements. Properly qualified hardware would have to be selected for the implementation, and software techniques (specification, design, verification and validation, etc.) that conform to NRC guidance would have to be used to implement the algorithms. Despite these hurdles, significant improvements in plant safety and operating margins could be achieved using this established, time-tested technique, if corresponding standards and regulatory guidance were developed, the implementations followed the standards and regulatory guidance carefully, and were planned and executed with a proper concern for safety and quality.

9 CONCLUSIONS

LQE/Kalman filtering provides a method whereby redundant measurements of safety-related process conditions in nuclear power plants can be estimated with greater precision than is available from individual instruments, while still maintaining the channel separation and integrity required for SR trip and engineered safety systems. The methods are computationally efficient and could be implemented in digital protection systems currently available on the market. Such implementations would provide significant reductions in measurement uncertainty, potentially providing improved SR protection and greater operating margins to nuclear power plants.

10 REFERENCES

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