Real-Time Fault Detection and Transient Identification Using Extended Kalman Filters

Jason G. Pickel and Daniel G. Cole
Department of Mechanical Engineering and Materials Science
University of Pittsburgh
3700 O'Hara Street, Pittsburgh, PA
jgp2@pitt.edu; dgcole@pitt.edu

ABSTRACT

In this paper, real-time fault detection and transient identification are studied with adaptive estimation methods, in particular parallel processing methods by implementing a bank of extended Kalman filters. Adaptive estimation is combined with probabilistic risk assessment to allow for real-time continuous monitoring of initiating events. The approach provides a probability of an initiating event happening. Real-time monitoring of initiating events can reduce labor demand. Each extended Kalman filter is configured to operate at a single configuration, either at normal operation or a faulted state. Fault detection is based on calculating a likelihood function for each extended Kalman filter to determine the posteriori conditional probability that the system configuration is the true configuration. The transient identification is determined by performing a multivariate statistical analysis on each extended Kalman filter to provide a posterior probability assessment that the reactor states and its dynamics are within the allowable limits. The probability distribution being outside the allowable limits indicate a transient has occurred. The combination of parallel processing methods with the multivariate statistical analysis for transient identification allows for the operator to perform transient identification with having a high level of certainty of the current system configuration. We demonstrate the real-time fault detection and transient identification on a simple reactor example to determine the reactor power, reactor temperature, and coolant temperature in the presence of various faults.

1 INTRODUCTION

The economic viability of small modular reactors require the reduction in labor demand. The labor demand can be reduced by increasing the level of automation in the processes for the reactor operations, fault detection, and transient identification. In addition to reducing labor demand, operators want to know current risk-assessment of the entire plant. The ability to use automation methods and provide a real-time probability analysis to assess the current state of the nuclear plant yields to the concept of probability risk assessment (PRA).

PRA is a methodology that describes the risk profile propagation from an initiating event (IE) to the ultimate consequence (UC). An initiating event may result in an incident (IN), which may lead to an accident (AC) and possibly to off-site release of radiation (OR) and ultimate consequences (UC). The incident state determines the type of emergency response necessary, and the emergency response directly determines the accident state. Based on the accident state, the accident may lead to containment failure (CF), which combined with the accident characteristics may lead to off-site release of radiation. The risk profile propagates from the IE to the UC using a Bayesian network, as shown in figure 1.

The Bayesian network describes the propagation of probabilities from the initiating event (IE) to the ultimate consequences (UC). The method and machinery of PRA is very effective at making the probability-based relationship between initiating events and ultimate consequences. Often an a priori probability of a particular initiating event is assumed, e.g., the chance of a sensor failing is 10^{-4}. These a priori probabilities can let engineers and decision makers assess designs on a risk basis; however, they have little meaning during the real-time operation of a plant. In that context, the operators and decision makers are more interested in whether a fault has occurred or not, but making such an assessment can be difficult or time consuming. In many situations, the operator must make an a posterior assessment that considers the data at hand, and apply a degree of belief to that assessment, e.g., I am 90 % sure the the device has failed.

In this paper, we present an approach we are calling *real-time risk assessment* which provides a real-time probability analysis for fault detection and transient detection. For fault detection, the approach provides the probability of a particular fault conditioned on the data that has been collected to that point. Thus, the a priori probabilities used to characterize initiating events can be replaced by a posteriori probabilities that account for the real-time response of the system. For transient detection, the approach provides the probability of a particular transient conditioned on the data that has been collected by the plant. Moreover, those a posteriori probabilities can be used with existing PRA analysis to determine risk in real-time.

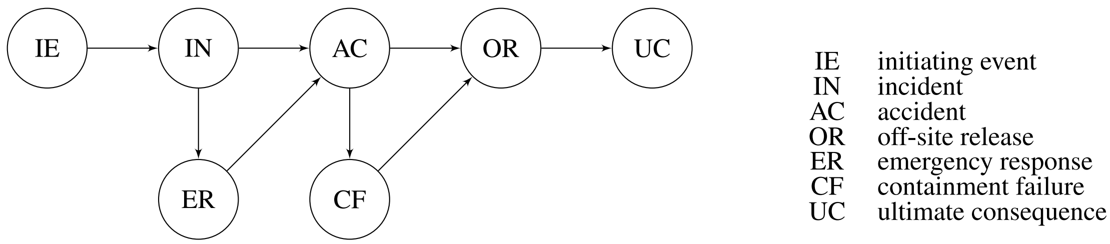

Figure 1: The risk profile propagates from the IE to the UC using a Bayesian network.

The real-time probability evaluations are focused on conditional probability and conditional cumulative distribution. The evaluations are computed in the context of combining fault identification and transient identification with adaptive estimation. We present the theory of the approach, as it would be used with a bank of extended Kalman filters (EKFs). Then we present a simplified reactor example to determine reactor power, core and coolant temperatures in the presence of various faults and transients.

2 PROBABILITY EVALUATION FOR FAULT DETECTION

Real-time fault identification requires determining the state of the system (its parameter configuration) when the signal model is unknown. The question can be answered using the concept of adaptive estimation from the field of system and control theory.

We will assume the true system parameter configuration is θ_i, with θ_i belonging to a finite discrete set $\Theta = \{\theta_1, \ldots, \theta_N\}$. The set Θ describes the system's parameter configuration at nominal conditions and at faulted conditions. The problem of determining the system's parameter configuration (achieving fault detection) is an adaptive estimation problem, which can be solved using parallel processing methods [6, 7] and sequential Monte Carlo methods [4]. Here we provide the architecture to use parallel processing methods to determine the system state and to achieve fault detection. Parallel processing methods are attractive because they can be implemented using parallel computing architectures, which are now well established, inexpensive, and easy to construct and expand.

The parallel processing method is described as follows: each $\theta_i \in \Theta$ has a known or assumed apriori probability $p(\theta_i)$. The true parameter configuration is estimated using a bank of N state observers, with the ith state observer designed on the assumption $\theta = \theta_i$, as shown in figure 2.

The choice of state observer is appropriate to the problem: Kalman filters for linear stochastic systems, extended Kalman filters and Unscented Kalman filters for non-linear systems with near Gaussian statistics, and particle filters for general non-linear/non-Gaussian systems [4, 2, 3].

The chosen observer is driven by noisy signal measurements, with the ith estimator yielding the conditional state-estimate $\hat{x}_{k|k-1,\theta_i}$. The conditional mean estimate $\hat{x}_{k|k-1}$ is generated by the weighted sum of the conditional state-estimates,

$$\hat{x}_{k|k-1} = \sum_{i=1}^{N} \hat{x}_{k|k-1,\theta_i} p(\theta_i|Z_{k-1}) \tag{1}$$

with the weighting coefficient being the posteriori probability $p(\theta_i|Z_{k-1})$ of each observer. The posteriori probability is updated recursively using the noisy signal measurements $Z_k = \{z_k, \ldots, z_0\}$ and the state estimate of the ith state observer. For each observer, its a posteriori probability $p(\theta_i|Z_k)$ is determined recursively from its a priori probability $p(\theta_i)$ using Bayes' rule,

$$p(\theta_i|Z_k) = \frac{p(Z_k,\theta_i)}{p(Z_k)} = \frac{p(Z_k|\theta_i)}{p(Z_k)} p(\theta_i) = \frac{p(Z_k|\theta_i)}{\sum_{i=1}^{N} p(Z_k|\theta_i) p(\theta_i)} p(\theta_i). \tag{2}$$

where $p(Z_k|\theta_i)$ is the likelihood function to be determined. The recursive calculation of these quantities is the advantage of combining parallel processing method with extended Kalman filters. The recursive calculation is based on Bayes' rule [1]:

$$p(\theta_i|Z_k) = \frac{p(z_k, Z_{k-1}, \theta_i)}{p(z_k, Z_{k-1})} = \frac{p(z_k|Z_{k-1},\theta_i)}{\sum_{i=1}^{N} p(z_k|Z_{k-1},\theta_i) p(\theta_i|Z_{k-1})} p(\theta_i|Z_{k-1}), \tag{3}$$

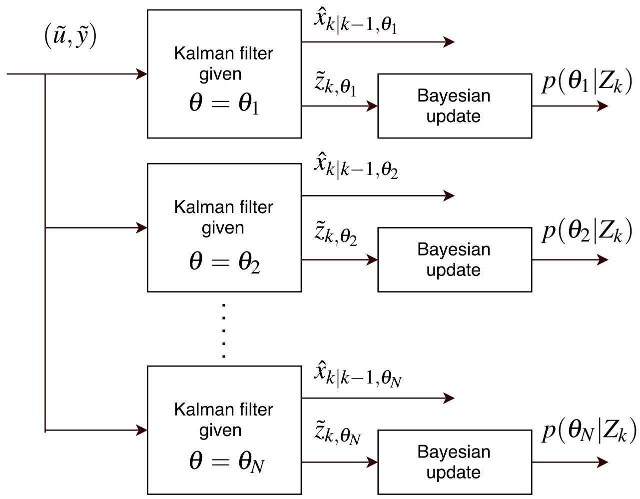

Figure 2: A bank of parallel filters can be combined with a Bayesian update to provide the conditional probability, $p(\theta_i|Z_k)$, that a particular fault has occurred.

where the denominator of (3) is a normalizing constant. The crucial problem is the calculation of the likelihood $p(z_k|Z_{k-1}, \theta_i)$. The calculation of $p(z_k|Z_{k-1}, \theta_i)$ is readily implementable when using Gaussian noise models. The combination of parallel processing methods with EKFs makes the implementation of real-time PRA possible.

The Kalman filter is a minimum variance estimator. Here, the various forms of the Kalman filter can be used interchangeably: linear, extended, and unscented. The EKF equations are given for a general nonlinear system [1],

$$\begin{aligned} x_{k+1} &= f_k(x_k) + \gamma_k(x_k)u_k + g_k(x_k)w_k, \\ z_k &= h_k(x_k) + v_k. \end{aligned}$$

The process noise w_k and measurement noise v_k are white, and their statistics are

$$\begin{aligned} E\{w_k\} &= 0, \qquad E\left\{w_k w_k'\right\} = Q_k, \\ E\{v_k\} &= 0, \qquad E\left\{v_k v_k'\right\} = R_k. \end{aligned}$$

The following matrices are used in the EKF equations:

$$F_k = \left.\frac{\partial f_k(x)}{\partial x}\right|_{x=x_{k|k}}, \quad H_k = \left.\frac{\partial h_k(x)}{\partial x}\right|_{x=x_{k|k}}, \quad \Gamma_k = \gamma_k(x_{k|k}), \quad G_k = g_k(x_{k|k}).$$

The EKF equations are then

$$\begin{aligned} \text{innovation} \qquad & \tilde{z}_k = z_k - h_k(\hat{x}_{k|k-1}) \\ \text{innovation covariance} \qquad & \Omega_k = H_k' \Sigma_{k|k-1} H_k + R_k \\ \text{Kalman filter gain} \qquad & L_k = \Sigma_{k|k-1} H_k \Omega_k^{-1} \\ \text{state predictor} \qquad & \hat{x}_{k+1|k} = f_k(\hat{x}_{k|k}) + g_k(\hat{x}_{k|k})u_k \\ \text{state corrector} \qquad & \hat{x}_{k|k} = \hat{x}_{k|k-1} + L_k \tilde{z}_k \\ \text{covariance predictor} \qquad & \Sigma_{k+1|k} = F_k \Sigma_{k|k-1} F_k' + G_k Q_k G_k' \\ \text{covariance corrector} \qquad & \Sigma_{k|k} = \Sigma_{k|k-1} - \Sigma_{k|k-1} H_k \Omega_k^{-1} H_k' \Sigma_{k|k-1} \end{aligned}$$

The likelihood function $p(z_k|Z_{k-1},\theta_i)$ is calculated for each EKF using the current measurement through the statistics of its innovation. The innovation is described by

$$\tilde{z}_{k|\theta_i} = z_k - \hat{z}_{k|\theta_i}, \tag{4}$$

where z_k is the current measurement, and the innovation covariance is

$$\Omega_{k|\theta_i} = E\left\{\tilde{z}'_{k|\theta_i}\tilde{z}_{k|\theta_i}\right\}. \tag{5}$$

Then, for each EKF, the likelihood function is [1]

$$p(z_k|Z_{k-1},\theta_i) = \frac{1}{(2\pi)^{p/2}}\left|\Omega^{-1}_{k|\theta_i}\right|^{1/2}\exp\left\{-\tfrac{1}{2}\tilde{z}_{k|\theta_i}\Omega^{-1}_{k|\theta_i}\tilde{z}_{k|\theta_i}\right\}, \tag{6}$$

and the a posteriori conditional probability is

$$\begin{aligned} p(\theta_i|Z_k) &= \frac{1}{C}p(z_k|Z_{k-1},\theta_i)p(\theta_i|Z_{k-1}) \\ &= \frac{1}{C}\left[\frac{1}{(2\pi)^{p/2}}\left|\Omega^{-1}_{k|\theta_i}\right|^{1/2}\exp\left\{-\tfrac{1}{2}\tilde{z}_{k|\theta_i}\Omega^{-1}_{k|\theta_i}\tilde{z}_{k|\theta_i}\right\}\right]p(\theta_i|Z_{k-1}) \end{aligned} \tag{7}$$

The constant C is a normalizing constant to satisfy the requirement $\sum_{i=1}^{N} p(\theta_i|Z_k) = 1$.

When the system's true parameter configuration is θ_i, the EKF configured to $\theta = \theta_i$ has its expected value of the innovation be zero, and its likelihood function and a posteriori conditional probability be close to one. The EKFs not configured to $\theta = \theta_i$ have their expected value of the innovation be a nonzero value, and their likelihood functions and a posteriori conditional probabilities be close to zero.

3 PROBABILITY EVALUATION FOR TRANSIENT DETECTION

Transients occur due to system faults and performing real-time transient identification requires determining if the time rate of change of the desired variable exceeds a threshold. Transient detection is addressed with multivariate cumulative distribution function (MCDF) analysis. The MCDF analysis is performed on each EKF in the filter bank because the system's parameter configuration must be known a priori to a transient happening.

We assume the transient occurs in the desired variable (may be a system state) q. The transient causes $\dot{q}$ to be very large. A transient occurs if $\dot{q}$ exceeds the threshold b. A variable $\dot{q}$ has a known multivariate joint normal probability density based on the EKF statistics,

$$p_k(\hat{x}) = \frac{1}{(2\pi)^{n/2}|\Sigma_k|^{1/2}}\exp\left((\hat{x}-\mu_k)^T\Sigma_k^{-1}(\hat{x}-\mu_k)\right), \tag{8}$$

where the expected value is $\mu_k = E[\hat{x}_{k|k}]$ and the variance is $\Sigma_k = \Sigma_{k|k}$. The statistical quantities of $\dot{q}$ are determined using the statistical characteristics of the EKF configured to the true parameter configuration. We use p_k to calculate the probability of the threshold being exceeded using a MCDF analysis.

The MCDF analysis provides the probability of $\dot{q}$ exceeding the threshold. The probability $P_k(-b \leq \dot{q} \leq b)$ describes $\dot{q}$ not exceeding the threshold and is determined using the joint probability density of the EKF statistics,

$$P_k(-b \leq \dot{q} \leq b) = \sum_{\dot{q}\in Q} p_k(\hat{x}). \tag{9}$$

where Q is the subset of the state estimate that satisfies $|\dot{q}| < b$. Then, $\dot{q}$ exceeds the threshold with the probability

$$P_k(\dot{q} < b \bigcup \dot{q} > b) = 1 - P_k(-b \leq \dot{q} \leq b). \tag{10}$$

The combination of the EKF filter bank and the multivariate cumulative distribution analysis allows real-time transient detection when system faults occurs and makes the implementation of real-time PRA possible.

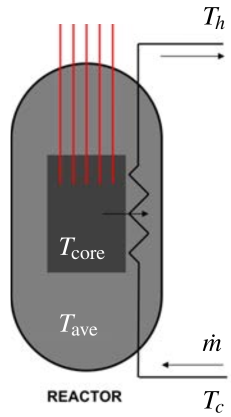

Figure 3: A schematic of the reactor simulated to demonstrate the real-time PRA method.

4 SIMPLE REACTOR EXAMPLE

We demonstrate the performance of parallel processing methods for fault and transient detection on a 4-state model of a reactor. The system schematic is shown in figure 3, and is composed of a nuclear reactor and a heat exchanger from core temperature to the coolant temperature. Delayed neutrons are approximated with a 1-group model, The reactor dynamics are characterized by the point kinetics equations,[5]

$$\dot{n} = \frac{\rho - \beta}{\Lambda} n + \lambda c \tag{11}$$

$$\dot{c} = -\lambda n + \frac{\beta}{\Lambda} c \tag{12}$$

where n is the neutron generation and is normalized to one for full power, and c is the delayed neutron state. The quantity β is the delayed neutron fraction, Λ is the neutron lifetime, λ is the decay constant for delayed neutrons, and ρ is reactivity. The reactivity is temperature dependent

$$\rho = \alpha(T_{\text{core}} - T_0),$$

where α is the (negative) temperature reactivity coefficient, T_{core} is the average core temperature, and T_0 is a reference reactor core temperature.

A heat balance on the heat exchanger yields the dynamics describing the reactor core and coolant temperatures being

$$m_1 c_1 \dot{T}_{\text{core}} = P_0 n - UA\left(T_{\text{core}} - T_{\text{ave}}\right) \tag{13}$$

$$m_2 c_2 \dot{T}_{\text{ave}} = UA\left(T_{\text{core}} - T_{\text{ave}}\right) - \dot{m} c_2 (T_h - T_c) \tag{14}$$

where T_{ave} is the average coolant temperature, $m_{1,2}$ is the thermal mass of the core or coolant respectively, $c_{1,2}$ is a corresponding specific heat, UA is the overall heat transfer coefficient, P_0 is the rated power of the reactor, and $\dot{m}$ is the mass flow of coolant through the reactor. The overall heat transfer coefficient UA and the cold-leg temperature T_c are assumed to vary randomly. These parameters are modeled as

$$UA = \overline{UA}\left(1 + w_1\right) \tag{15}$$

$$T_c = \bar{T}_c + w_2 \tag{16}$$

where w_1 and w_2 are zero-mean Gaussian white noise with spectral densities S_{w1} and S_{w2} respectively. The nominal coolant temperature $\bar{T}_c$ is assumed given.

The average coolant temperature is described by

$$T_{\text{ave}} = \tfrac{1}{2}(T_h + T_c), \tag{17}$$

where T_h is the hot-leg temperature. Since the temperature T_c is known, T_h is a function of the state T_{ave},

$$T_h = 2T_{\text{ave}} - T_c. \tag{18}$$

The updated heat balance equations are

$$m_1 c_1 \dot{T}_{\text{core}} = P_0 n - \overline{UA}(T_{\text{core}} - T_{\text{ave}})(1 + w_1) \tag{19}$$
$$m_2 c_2 \dot{T}_{\text{ave}} = \overline{UA}(T_{\text{core}} - T_{\text{ave}})(1 + w_1) - 2\dot{m}c_2(T_{\text{ave}} - \bar{T}_c - w_2) \tag{20}$$

with the constant being defined as

$$K = \frac{P_0}{m_1 c_1}, \quad \omega_1 = \frac{UA}{m_1 c_1}, \quad \omega_2 = \frac{UA}{m_2 c_2}, \quad \text{and} \quad \omega_3 = 2\frac{\dot{m}}{m_2}.$$

Then, the heat balance equations are simpplified to

$$\dot{T}_{\text{core}} = Kn - \omega_1(T_{\text{core}} - T_{\text{ave}})(1 + w_1) \tag{21}$$
$$\dot{T}_{\text{ave}} = \omega_2(T_{\text{core}} - T_{\text{ave}})(1 + w_1) - \omega_3(T_{\text{ave}} - \bar{T}_c - w_2) \tag{22}$$

It is convenient to express the reactor equations and heat balance equations in state-space form:

$$\dot{x} = f(x) + g(x)w, \tag{23}$$
$$z = h(x) + v, \tag{24}$$

where the state is $x = [n,\, c,\, T_{\text{core}}, T_{\text{ave}}]'$ and the measurements are $z = [n,\, T_{core}]'$. The process noise $w = [w_1,\, w_2]'$ and the measurement noise v are zero mean Gaussian white noise with spectral densities $S_w = \text{diag}\{S_{w1}, S_{w2}\}$ and $S_v = \text{diag}\{S_{v1}, S_{v2}\}$ respectively.

4.1 Discrete-time Model

The continuous-time system equations given above are implemented in discrete-time using a one-step, fourth-order Runge-Kutta method. The resulting update equations can be most generically represented in discrete-time as

$$x_{k+1} = f(x_k) + g(x_k)w_k \tag{25}$$
$$z_k = h(x_k) + v_k \tag{26}$$

where, for a sufficiently small time-step τ, the discrete-time process and measurement noise are zero mean with covariance

$$E\left\{w_k w_k'\right\} = Q = S_w\tau = \text{diag}\{S_{w1}\tau, S_{w2}\tau\} \tag{27}$$
$$E\left\{w_k w_k'\right\} = R = S_v\tau = \text{diag}\{S_{v1}\tau, S_{v2}\tau\} \tag{28}$$

This discrete-time model is directly amenable to the extended Kalman filter process outline previously.

4.2 Simulations

Simulations are performed to demonstrate the methods outlined above. A model of the respective open-loop systems are constructed in Simulink. The discrete-set of all possible parmeter configurations are:

- Fault 0 (θ_1): Nominal operating conditions (no fault has occurred).
- Fault 1 (θ_2): $\overline{UA}$ is reduced to 75% of its nominal values.
- Fault 2 (θ_3): $\overline{UA}$ is reduced to 50% of its nominal values.
- Fault 3 (θ_4): S_{v1} is increased by a factor of 10.

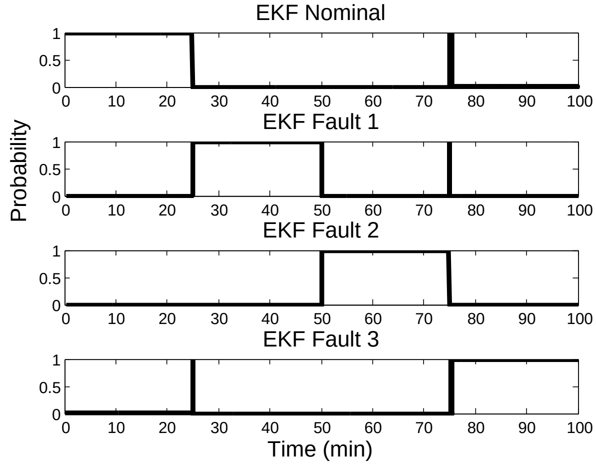

Figure 4: The posteriori probability analysis for each extended Kalman filter. When the system is operating at a particular fault, the extended Kalman filter operating at that fault condition yields the most likely system configuration because its probability is close to one.

The faults were introduced in the following way: The system initially operates at the nominal condition (θ_1) for 0 min to 25 min; the parameter configuration changed to θ_2 for 25 min to 50 min; the parameter configuration changed to θ_3 for 50 min to 75 min; the parameter configuration changed to θ_4 for 75 min to 100 min.

The current fault condition is determined using the parallel filter technique outlined previously. Each extended Kalman filter is configured operate at one of the faulted conditions. The filter bank produces both a state estimate, $\hat{x}_{k,\theta_i}$, and the conditional probability, $p(\theta_i, Z_k)$, for each extended Kalman filter.

A fault causes transients in the reactor state. We are interested in the transient occurring in the power state, which has the derivative being (11)

$$\dot{n} = \frac{\rho - \beta}{\Lambda} n + \lambda c. \tag{29}$$

The variable $\dot{n}$ is a random variable of the sum of two random variables n and c. The random variables n and c have a jointly normal probability density with the statistics being the statistics of the EKF that is configured to operateat the current fault condition. The joint probability density being normal is the result of the Gaussian noise statistical assumption in the EKF formulation. Then, the joint probability density is used to determine the probability of $\dot{n}$ exceeding the threshold b.

5 RESULTS AND DISCUSSION

The conditional probability of each EKF is shown in figure 4. The system operates at nominal operating conditions for the first 25 min, and results in a near close to one conditional probability for the EKF configured to fault zero. The transition in the system faults (e.g., fault zero to fault one, fault one to fault two, and fault two to fault three) result in transients in the conditional probabilities. The transients quickly decays to show the filter bank has converged to the EKF configured to operated at the current fault.

The parallel processing method provides the state estimate using the weighted sum of the conditional state estimate of each EKF. The response of the true power and its expected value are shown in figure 5; the response of the core temperature, coolant temperature, and their expected values are shown in figure 6. The filter bank provides an unbiased state estimate for the case when the system is configured to operate at nominal operating conditions (first 25 min). The presence of the sequential system faults (occurring at 25 min, 50 min, and 75 min) has no affect on the bias of the state estimate when the filter bank has one filter that is configured to operate at the faulted condition.

A transient is detected by combining the conditional probability of the filter bank (figure 4) with the MCDF analysis of the filter bank. The response of the MCDF analysis on the filter bank provides a probability a transient has occurred, as shown in figure 7. The system is initially configured to operate at nominal operating conditions (fault 0 for the first 25 min), and the MCDF analysis for the EKF yielded a probability 0.43 of a transient occurring. The MCDF analysis for the other EKFs are ignored because of the low confidence the system is operating at one of these faulted conditions. At 25 min, the parameter configuration is changed to θ_2 (fault 1) and the conditional probability analysis shows there is high confidence the system is operating at fault 1. The corresponding MCDF analysis for that EKF shows there is high confidence a transient has occurred between 25 min to 40 min, then the MCDF analysis yields a probability 0.37

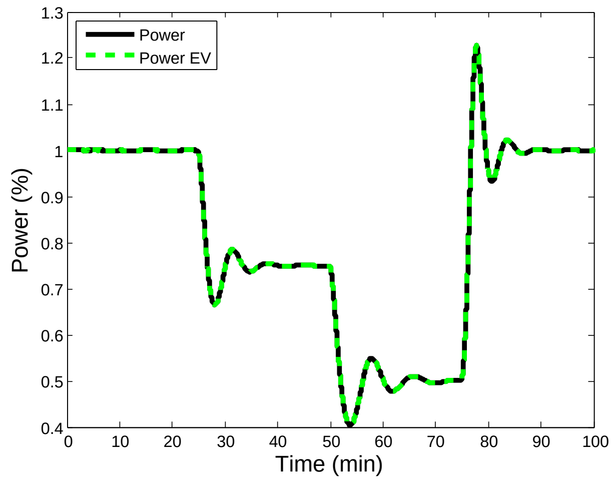

Figure 5: The response of the system's power and its expected value using a bank of extended Kalman filters.

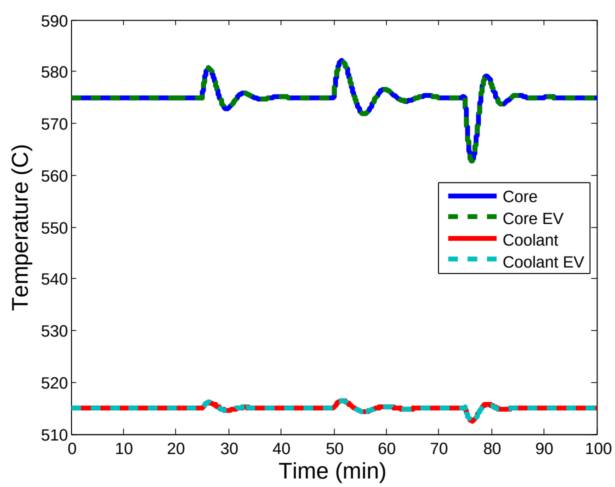

Figure 6: The response of the core temperature, coolant temperature, and their expected values using a bank of extended Kalman filers.

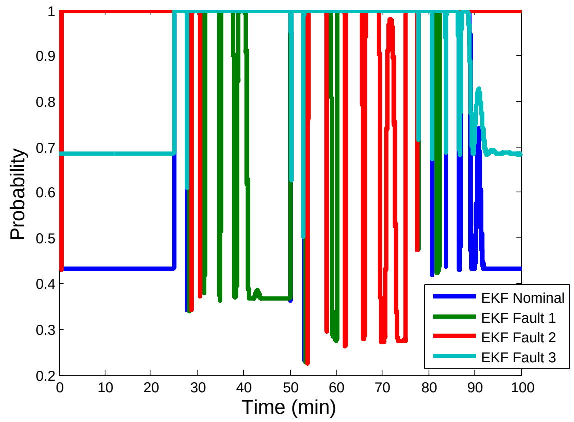

Figure 7: The response of the MCDF analysis on each EKF to quantify the probability a transient has occurred in the reactor power.

of a transient occurring between 40 min to 50 min. The parameter configuration is changed to operate at θ_3 (fault 2) between 50 min to 75 min. There is high confidence the EKF configured to fault 2 is the true system configuration. The MCDF analysis for that EKF shows there is confidence a transient occurs throughout the time interval. The parameter configuration is changed to θ_4 (fault 3) for the last 25 min. The MCDF analysis for the EKF configured to fault 3 shows there is high confidence a transient occurred between 75 min to 89 min, then probability reduces to 0.69 during the remaining part of the time interval.

6 CONCLUSIONS

The economic viability of small modular reactors require the reduction in labor demand. This paper explored the use of automation methods and real-time probability analysis to assess the current system state in terms of a probability risk assessment (PRA). We used parallel processing methods, filter bank of EKF, for fault detection. Each filter is figured to a parameter configuration, and has a Bayesian update peroformed to calculate its real-time conditional a posteriori probability. A fault causes transients in a desired value and transient occurs when the time derivative of the desired value exceeds a certain value. In addition, real-time transient detection was performed using the statistics obtained from the EKF.

7 REFERENCES

1. B. D. O. Anderson and J. B. Moore. *Optimal Filtering*. Dover Publications, INC., 2005.
2. M. S. Arulampalam, S. Maskell, N. Gordon, and T. Clapp. A tutorial on particle filters for online nonlinear/non-gaussian bayesian tracking. *IEEE Trans. Sig. Proc.*, 50(2):174–188, 2002.
3. O. Brun, V. Teuliere, and J.-M. Garcia. Parallel particle filtering. *Journal of Parallel and Distributed Computing*, 62(7):1186–1202, 2002.
4. O. Cappe, S. J. Godsill, and E. Moulines. An overview of existing methods and recent advances in sequential monte carlo. *Proc. of the IEEE*, 95(5):899–924, 2007.
5. D. L. Hetrick. *Dynamics of Nuclear Reactors*. American Nuclear Society, Inc., 1993.
6. D. G. Lainiotis. Optimal adaptive estimation: Structure and parameter adaption. *IEEE Trans. Autom. Contr.*, 16(2):160–170, 1971.
7. P. K. S. Tam and J. B. Moore. Adaptive estimation using parallel processing techniques. *Computers & Electrical Engineering*, 2(2-3):203–214, 1975.