# A LARGE SCALE SYSTEM APPROACH FOR STABILITY ANALYSIS OF MULTI-UNIT SMR PLANTS

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**ABSTRACT**

In this paper we present a framework based on large scale system theory for analyzing the stability of coupled advanced high temperature molten salt small modular reactors.

It has been suggested that small modular reactors be coupled through a common molten salt energy repository, called a salt vault. Using vector Lyapunov functions the connective stability of multiple reactors coupled through a salt vault can be guaranteed. In this theory, scalar Lyapunov functions are assigned to portions of the composite system state space. Functions are constructed for each small modular reactor (SMR), each Brayton cycle, and the salt vault. These functions are then aggregated, becoming components of a vector Lyapunov function. If this vector function possesses special properties, the composite system is stable.

Bounds on the connective stability of a multi-reactor plant can be found in terms of a free parameter, in this case the mass of molten salt in the salt vault. Examining bounds on the connective stability allows one to use control theory as a tool for making performance and economic design decisions related to the size of the salt vault. This technique can produce a constraint that is used in a design optimization problem for finding the smallest-possible salt vault that ensures stability. It can also be used as an indicator of interconnection and its effect on system behavior. This, in turn, could be used to guide multi-unit control system design to ensure that highly coupled systems behave favorably. The theory for multi-unit control and operation is discussed.

*Key Words:* Lyapunov stability theory, vector lyapunov functions, small modular reactors, nuclear instrumentation and controls

## 1 INTRODUCTION

The vision for SMRs goes beyond large single-unit power generation to multiple small units at one site supplying heat to gas turbines and the co-generation for industrial needs Ref. [1, 2, 3]. Furthermore, it has been established that an increased level of automation will likely be necessary Ref. [3, 4]. The autonomous control of multiple, coupled, SMRs is a challenge. In this research, we analyze the stability of independently controlled, coupled Small Modular Advanced High Temperature Reactors (SmAHTR) Ref. [5] using basic system theoretic techniques and then extend into some approaches and techniques from large scale system theory Ref. [6, 7]. Casting the coupled multi-reactor control problem into a framework germane to large scale system theory offers several advantages:

1. Reduction of dimensions and the parameterization of key plant interconnections for stability analysis.

2. Allows for system designers and analysts to test the connective stability of multiple reactor configurations, controller designs, and operating points without disturbing other regions of the state space. This enables somewhat of a "plug and test" approach to evaluating connective stability.

3. Provides an entry point for multi-module SMR control problems into the rich field of large scale system theory.

These advantages will be illustrated and discussed in this paper. We will begin by describing the controlled SmAHTR system model that is being cast into this framework.

## 2 SMALL MODULAR ADVANCED HIGH TEMPERATURE REACTOR MODEL

The authors created a real-time executable simulation of four SmAHTR molten salt reactors, coupled through a common energy repository called a salt vault Ref. [5]. Lumped-parameter modeling strategies were used. For example, core kinetics are modeled using a spatial kinetic method including neutronics, temperatures, and poisoning/feedback effects. Each SmAHTR reactor is controlled in two principal ways: 1) Power regulation: a triplet of counterflow heat exchangers independently track reference power inputs; and 2) Core outlet temperature regulation: control rods are actuated to track a core outlet temperature setpoint. In this real-time simulation testbed, the transient behavior of the entire SmAHTR system can be monitored and controlled in real time. Changes to controller reference inputs, changes to system parameters, and sensor and actuator faults can be modified on a parallel computing kernel. A 3-D rendering of the SmAHTR is shown in figure 1. A 2-D schematic of one SmAHTR reactor is shown in figure 2. The dynamic behavior of most physical systems can be described using linear or nonlinear ordinary differential equations. For example, coupled core spatial kinetic/thermal hydraulic dynamics can be described using nonlinear ordinary differential equations Ref. [8]. The full system simulation that the authors codified in Matlab/Simulink in the Instrumentation and Controls Laboratory at the University of Pittsburgh simulates and controls a fully-nonlinear model of this plant. Well-known nuclear power plant system modeling techniques were used, such as those developed by Ref. [8, 9, 10]. For this analysis, we have linearized any nonlinear subsystem models about operating points within the "normal" operating regime. This linearization is done so that we can find the stability of the composite system using linear large scale system techniques. We mention that this theory is applicable to nonlinear systems as well, and has been rigorously developed and applied to nonlinear systems Ref. [11, 6, 12]. As an introduction to some of these techniques and how they can enhance our ability to analyze these very large, interconnected dynamic systems, we will present the linear version of large scale system theory.

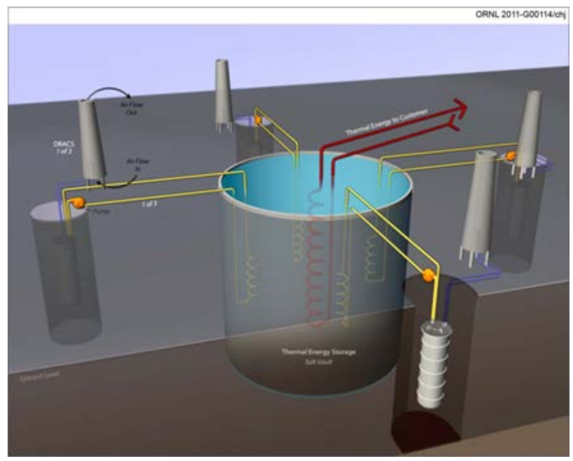


**Figure 1: 3-D Rendering of coupled SmAHTRs Ref. [5] S.R. Greene et. al., Pre-conceptual design of a fluoride-salt-cooled small modular advanced high-temperature reactor(SmAHTR). Technical Report ORNL/TM-2010/199, Oak Ridge National Laboratory, 2010.**

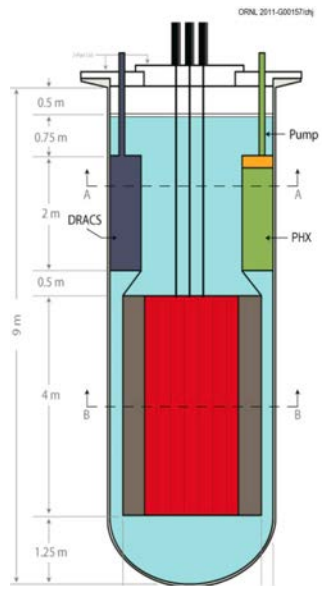


**Figure 2: 2-D SmAHTR reactor schematic. Labeled are the primary heat exchanger (PHX) and direct reactor auxiliary cooling system (DRACS). The DRACS system causes a minor (1%) power loss during normal operation. During normal operation and in our model, the DRACS contribute a memoryless power loss. Ref. [5] S.R. Greene et. al., Pre-conceptual design of a fluoride-salt-cooled small modular advanced high-temperature reactor(SmAHTR). Technical Report ORNL/TM-2010/199, Oak Ridge National laboratory, 2010.**

Large scale system theory attempts to formalize the natural way that engineers think about large systems as a collection of linked, or coupled, smaller subsystems, whose composite behavior constitutes the dynamics of the large scale system. For this analysis, we have decomposed the SmAHTR plant shown in figures 1 and 2 into reactor subsystems, the salt vault, and Brayton cycles. The reactor systems are comprised of spatial kinetics/thermal hydraulics (core dynamics), downcomer dynamics, three parallel counterflow heat exchangers (PHX1, PHX2, PHX3), and salt vault mixing dynamics. A simple schematic of these groupings is shown in figure 3. Within figure 3 we also indicate energy flow using blue and red arrows in this system. What is important to note about each reactor system is that the spatial kinetics/thermal hydraulics and primary heat exchangers are closed-loop systems. That is, controllers have been designed for the reactor kinetics/thermal hydraulics and each of the three primary heat exchangers. The core outlet temperature of the spatial kinetics and thermal hydraulics subsystem is regulated using a proportional-integral (PI) control, and the power extracted from each PHX is regulated using linear-quadratic-integral

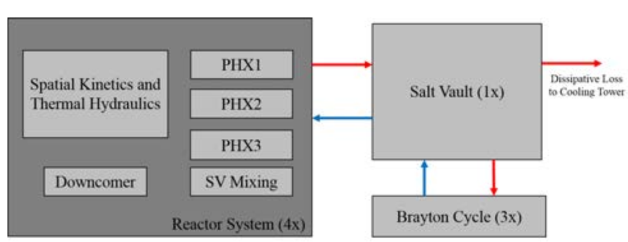


**Figure 3: Four reactor systems are comprised of subsystems as shown in this figure. Each reactor is coupled to the salt vault. Three Brayton cycles are modeled for electricity generation. Each Brayton cycle is connected to the salt vault.**

controllers which were designed around a linearized model of each PHX. The controller design details are not included in this paper. Rather, we show that *any* controller design for a reactor can be placed into this framework for analyzing connective stability.

Through linearization of these dynamic models and with controllers designed and implemented around the core models and primary heat exchangers we are able to use the generic state space representation of each dynamic system:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned} \tag{1}$$

where $x$ is our state vector, $u$ is our vector of inputs, and $y$ is our vector of outputs. State space models of subsystems can be easily aggregated by connecting the outputs of some systems with the inputs of downstream systems and vice versa. For one reactor model in this paper, we have $A \in \mathbb{R}^{70\times 70}$, meaning one reactor model is described by 70 ordinary differential equations. When all four reactors and all three Brayton cycles are coupled to the salt vault we have a composite, large scale model that has 292 states. We shall refer to the full, composite system model using the quadruplet ($A_{\text{comp}}$,$B_{\text{comp}}$,$C_{\text{comp}}$,$D_{\text{comp}}$) as represented in Eq. (2), but will be primarily interested in studying $A_{\text{comp}}$.

## 3 SYSTEM COUPLING

Figure 3 shows how subsystems are part of larger systems (PHX1, PHX2, PHX3, Spatial Kinetics and Thermal Hydraulics, etc.). These subsystems are naturally interconnected to form a larger, composite system. By representing these systems using traditional state space techniques, we can combine these systems with series, parallel, and feedback connections. These techniques were used to form the composite closed-loop plant, comprised of all four SmAHTR reactors, three Brayton cycles, and salt vault.

To use large scale system theory we must judiciously model subsystem *interconnections*. This is a topic of ongoing research, and the general theory and approach will be discussed herein. This research begins to analyze the stability of the composite closed-loop system as a function of subsystem interconnectivity through the salt vault. To this end, the system model must be cast into a form that is germane to the field of large scale system theory for isolating these particular subsystem interconnection parameters in this composite system. Interconnection strength, in our case, becomes a function of the mass, or volume, of the salt vault. A fundamental presentation of the large scale system theory representation will be given later in this paper.

### 3.1 System Coupling, Stability, and Control

Any large system is, essentially, a collection of smaller interconnected subsystems. A nuclear power plant is an excellent manifestation of this definition. Due to the sheer size and complexity of a small modular reactor, it may be necessary to design controllers for subsystems with minor consideration to how these controlled subsystems interact with the systems that are downstream/interconnected. It is not this situation that we are going to examine directly, however. Large scale system theory and decentralized control techniques could be used to study one single reactor, but we are more interested in studying the connective stability of the controlled reactors with the salt vault and Brayton cycles. In this sense, we are examining connective stability at the *plant* level, and not the *system* level.

To this end, we can posit that controller design for the reactors can be approached in several ways:

1. Centralized controller design using any variety of methods Ref. [13, 14]. Centralized controller design implies that the entire reactor is considered as one composite system and a controller is designed that governs system behavior centrally, and through using all reactor state information simultaneously.

2. Decentralized control design techniques Ref. [6]. A decentralized approach only considers local subsystem dynamics: for example, the dynamics of other systems, such as the reactor core, are not used *directly* during primary heat exchanger controller design. Measurements taken with respect to core dynamics are not used for generating the primary heat exchanger automatic control inputs.

What is powerful in using large scale system techniques is that the exact control design method is not of significant consequence to our ability to test system connective stability. Now, controllers can be designed for reactors where one controller is inherently "more stable" than another. This situation could mean that one controlled reactor leads to a failed connective stability test, whereas another does not. The technique that we present herein is generalizable to any controller design for a reactor. Therefore, these techniques give us the ability to check the connective stability of multiple controller designs for multiple SmAHTR reactors across multiple operating regimes.

For one closed-loop reactor system it is easy to test that the system is stable by verifying that the reactor system dynamics matrix (call it $A_{\text{reactor}}$) has eigenvalues with negative real part. Later, we will show how Lyapunov functions can be used to condense this notion of stability down into a scalar variable used in testing for connective stability of the composite system. This condensation shows equivalence between a system with eigenvalues possessing negative real parts and one that satisfies the Lyapunov equation, which gives us a scalar expression for stability.

We stated previously that we are interested in testing the stability of the composite system as a function of the size of the salt vault. Without using large scale system theory, we know that we can use the full 292 state model to do this. Further, we are interested in examining opposite ends of this spectrum:

1. For the case where the volume of salt in the salt vault is very small, which will lead to strong coupling between reactor systems and energy sinks

2. For the case where the volume of salt in the salt vault is very large, which will essentially decouple the reactors. Charging of the salt vault would of course happen very slowly, but once "at capacity" the salt vault would buffer other plant systems from perturbing one another.

Across this spectrum of sizes for the salt vault, there are advantages and disadvantages related to system operation and economics. From an operational perspective, maximizing the volume of the salt vault would enable us to design stable, controlled reactors without much, if any, consideration to how these controlled reactors could influence one another. This large salt vault would serve as a buffer between these reactors that are providing energy, and would be acting as a low pass filter between reactors. That is, it would buffer the reactors experiencing "sharing" transients in temperature and flow fluctuations. However, economics would drive us to choose the size of this salt vault judiciously. An extremely large salt vault would come with great material and construction costs. A lumped-parameter schematic of the salt vault that shows how energy transfer occurs from reactor "sources" to Brayton cycle "sinks" is depicted in figure 4. In figure 4, one additional sink is shown to represent a cooling tower. In this paper, the cooling tower and salt vault are being modeled as one composite subsystem. As described previously, we have modeled a linearized version of this salt vault in the form Eq. (2). To illustrate how the volume of the salt vault can have the effect of "decoupling" plant systems, we examine the salt vault's A matrix, which we will call $A_{\text{salt}}$ where it has the form:

$$A_{\text{salt}} = \begin{bmatrix} * & 0 & 0 & 0 & * & 0 & 0 & 0 & 0 \\ 0 & * & 0 & 0 & * & 0 & 0 & 0 & 0 \\ 0 & 0 & * & 0 & * & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & * & * & 0 & 0 & 0 & 0 \\ \frac{*}{\tau_s} & \frac{*}{\tau_s} & \frac{*}{\tau_s} & \frac{*}{\tau_s} & \frac{*}{\tau_s} & \frac{*}{\tau_s} & \frac{*}{\tau_s} & \frac{*}{\tau_s} & \frac{*}{\tau_s} \\ 0 & 0 & 0 & 0 & * & * & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & * & 0 & * & 0 & 0 \\ 0 & 0 & 0 & 0 & * & 0 & 0 & * & 0 \\ 0 & 0 & 0 & 0 & * & 0 & 0 & 0 & * \end{bmatrix} \tag{2}$$

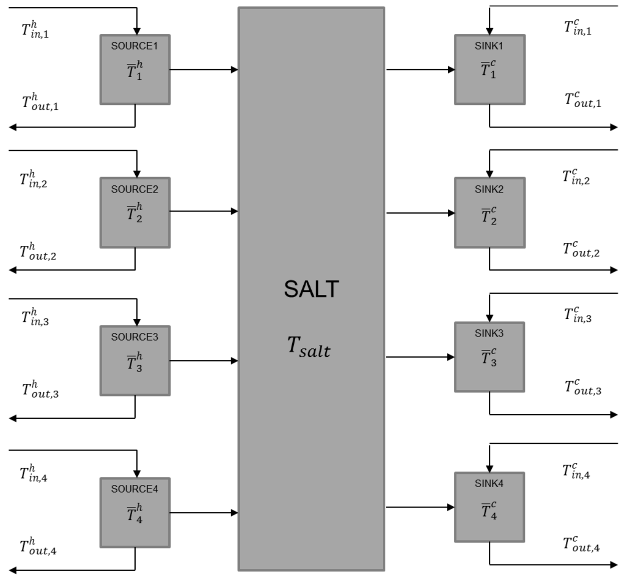


**Figure 4: Lumped parameter schematic of the SmAHTR salt vault. Shown are four energy sinks. In the SmAHTR model, three sinks are modeled as Brayton cycle subsystems. The other sink is an "ultimate" cooling tower sink. In this work, the cooling tower and salt vault are modeled as one subsystem.**

where

- $\tau_s = m_s c_s$. This represents the mass of salt and the specific heat of the molten salt, respectively.
- $*$ represents some nonzero entry in the salt vault dynamics matrix.

The structure of this matrix makes it apparent how the reactor subsystems and Brayton cycles couple through the salt in this salt vault. What is quickly seen is that as the mass of salt, $m_s \to \infty$ an entire row of the salt vault dynamics matrix approaches zero, having the effect of decoupling the system. This row describes the time derivative of the bulk salt temperature in the salt vault. Since this time derivative is zero, the salt vault temperature can only assume some static value and is no longer affected by source/sink interaction. Conversely, as $m_s \to 0$ this same row approaches infinity, which has the effect of strongly coupling the reactors to one another and the Brayton cycles. This basically creates a short circuit that couples all reactors and Brayton cycles to one another. This situation could result in undesirable system dynamics and may also lead to instability for some controlled reactor configurations. We will now get into discussing large scale system theory and techniques that have been developed for testing the connective stability of controlled subsystems. Then, we will discuss how the SmAHTR plant model is being fit into this framework.

## 4 LARGE SCALE SYSTEM THEORY

Large scale system theory provides an alternative way for finding the stability of an interconnected, decentrally-controlled dynamic system that reduces the dimensionality of the problem, remains applicable to fully nonlinear systems, allows for the parameterization of coupling terms for evaluating stability as a function of the strength of subsystem interconnections, and casts the system into a form that allows for global control law design. Some of these benefits will be demonstrated and discussed in this paper. We will first provide a brief explanation about how large scale systems are represented as interconnections of decentrally-modeled and controlled subsystems. Several texts have been written that discuss the theory, history, and techniques germane to this sub-field within system theory Ref. [6, 7, 11].

## 4.1 Large Scale System Representation

The fundamental form for describing interconnected subsystems in large scale system theory is given by:

$$\dot{x}_i = A_i x_i + B_i u_i^l + \sum_{j=1}^{N} A_{ij} x_j, \qquad i \in N \tag{3}$$

where

- $x_i$ represents the local system state vector.
- $N$ is the number of subsystems that comprise the composite system. For the SmAHTR system in this paper, $N = 8$.
- $A_i$, $B_i$ represent the local subsystem dynamics and input matrix for each subsystem, respectively.
- $A_{ij}$ is the interconnetion matrix that couples subsystem $i$ to subsystem $j$.
- $x_j$ represents the states local to subsystem $j$

And so by connecting the four reactor models, salt vault, and Brayton cycles of the SmAHTR reactor as depicted in figures 1, 2, and 3 we get $N = 8$. Subsystems are reactor #1, reactor #2, reactor #3, reactor #4, Brayton cycle #1, Brayton cycle #2, Brayton cycle #3, and the salt vault, totaling 8. The interconnections between these subsystems occur through the salt vault, as the salt vault is our energy repository. The strength of the interconnections can be related to the volume of the salt vault, making it a parameterizable variable for investigating composite system stability limits.

## 4.2 Stability Theory

### 4.2.1 General Lyapunov Stability Theory

In large scale system theory, stability is found using vector Lyapunov functions. The results of this theory will be presented here. The authors refer the reader to Ref. [6, 7, 11, 12] for more detailed discussions and derivations behind this theory. Vector Lyapunov theory is an extension of its scalar counterpart, which we review briefly.

Given a stable system $\dot{x}_i = A_i x_i$, which is a *closed-loop* subsystem of our SmAHTR model *with interconnection terms removed*, we know that this closed-loop system is stable if the real part of the eigenvalues of $A_i$ are strictly negative. Given a stable system matrix $A_i$, the Lyapunov equation is given by

$$A_i^T H_i + H_i A_i + G_i = 0 \tag{4}$$

where $H_i, G_i > 0$ (positive definite) and symmetric. A powerful result of Eq. (4) is that it can be solved for some unique positive definite matrix $H_i$ given some positive definite matrix $G_i$ and that the system $A_i$ is stable. Therefore, we define the Lyapunov function for this general system to be

$$V_i(x_i, t) = x_i^T H_i x_i \tag{5}$$

The Lyapunov function can be thought of as the "energy" present in the system. With judicious choice of $H_i$, the scalar value $x_i^T H_i x_i$ is exactly the energy in the $i$th subsystem. From Eq. (5) we see that if we take the total derivative of $v(x, t)$ along the trajectories of the system we get

$$\dot{V}_i(x_i, t) = x_i^T A_i^T H_i x_i + x_i^T H_i A_i x_i = -x_i^T G_i x_i \tag{6}$$

which shows that the system's energy is dissipating from the system since $G_i > 0$. Therefore, the term $-x_i^T G x_i$ can be thought of as the system's energy "dissipation" rate. Furthermore, since $x \in \mathbb{R}^{n\times 1}$, this quadratic function produces a scalar value, and shows that for *for any* values of our state vector $x_i \in \mathbb{R}^n$ energy will be dissipating from the system since $G$ is positive definite.

The key result for the Lyapunov equation given in Eq. (4) is that for some stable system described by $\dot{x}_i = A_i x_i$ we can find a unique positive definite Lyapunov function Eq. (5) by solving the Lyapunov equation Eq. (4) for any $G_i > 0$. This explains the basics of Lyapunov stability theory for a general, linear system.

Therefore, for our problem we must define positive definite matrices $G_i$'s for each of our eight subsystems. We note that each of these subsystems, when we solve for the subsystem Lyapunov functions, are devoid of the composite system interconnection terms described by the $A_{ij}$ matrices described in Eq. (3). Recall that the eight subsystems in this analysis are the following:

1. SmAHTR reactors #1, #2, #3, #4
2. Brayton cycles #1, #2, #3
3. Salt vault

In the next section, we discuss the results of vector Lyapunov theory and how its scalar counterpart, as discussed above, fits into its framework. Then, we will discuss how the SmAHTR plant system can be fit into this framework for stability analysis and control synthesis.

### 4.2.2 Vector Lyapunov Theory

Vector Lyapunov theory assigns scalar Lyapunov functions to portions of the composite system state space in such a way that each scalar function determines a desired stability property in a part of the composite state space where the other functions do not Ref. [6]. This was roughly illustrated in the preceding discussion on finding the unique scalar Lyapunov functions for individual subsystems while ignoring subsystem interconnections.

We must now address the subsystem interconnections, which are of principal interest in this paper. The interconnections are bounded by defining the terms $\zeta_{ij} = \lambda_M^{1/2}(A_{ij}^T A_{ij})$ where $\lambda_M^{1/2}(\cdot)$ denotes the square root of the maximum eigenvalue. The $A_{ij}$ terms describe the significant subsystem interconnection terms as shown in Eq. (3). By following the results derived in Ref. [6, 11] or Ref. [12, 7] we get a matrix, which we will call $W$, that is comprised of the following elements:

$$w_{ij} = \begin{cases} \frac{1}{2}\frac{\lambda_m(G_i)}{\lambda_M(H_i)} - \zeta_{ij}, & i = j \\ -\zeta_{ij}, & i \neq j \end{cases}$$

The matrix $W$ is square and is of dimension equal to the number of subsystems in our composite system. For the SmAHTR plant, this means that $W \in \mathbb{R}^{8\times 8}$. The $\lambda_m(\cdot)$ corresponds to the minimum eigenvalue, and the $\lambda_M(\cdot)$ corresponds to the maximum eigenvalue. Since $G_i$ and $H_i$ are positive definite and symmetric, all eigenvalues are both positive and purely real. The derivation with associated theorems and proofs behind forming this $W$ matrix is nontrivial and is a subject of several texts Ref. [6, 7, 11, 12]. This derivation is not the subject of this paper, rather, we are intrigued by the result that allows for us to *greatly reduce the dimension of the stability problem with a keen focus on stability as a function of interconnections*. What is stressed, however, is that a high-dimensional problem, such as that involved in finding the stability of coupled reactor systems, can be reduced down to one that is of a dimension equal to the number of subsystems in the model.

By Theorem 2.5 of Ref. [6] the composite system is stable if the matrix $W$ has special properties. Those properties happen to characterize an $M$-matrix, which is a type of matrix that appears frequently in economics and system theory. There are many equivalent conditions that characterize an $M$-matrix Ref. [15], and so we provide a few equivalent characteristics that describe $M$-matrices:

- Characteristic #1: All principal minors of the matrix are positive. This means that the determinant of submatrices obtained by deleting sets of corresponding rows and columns is positive.
- Characteristic #2: Every real eigenvalue of the matrix is positive.
- Characteristic #3: All leading principal minors of the matrix are positive.

Any of the checks above, and any of 37 others Ref. [15], can be used to verify these conditions. Clearly, the numerical effort behind checking these conditions is not too large and can be easily done with a desktop computer.

By inspection of the $w_{ij}$ elements we see that the subsystem interconnection terms ($A_{ij}$) terms are explicitly taken into account. These terms can therefore be parameterized for finding bounds. We can entertain this situation for a very simple case, where we have only two connected subsystems. Without deriving the equations of motion we can jump to the resulting $W$ matrix Ref. [6], which would look something like the following:

$$W = \begin{bmatrix} a-\zeta & -\zeta \\ -\zeta & a-\zeta \end{bmatrix} \tag{7}$$

where $a$ is the positive parameter that describes the ratio between the smallest eigenvalue of the positive definite matrix $G$ (describing the maximum rate of "dissipation" for each dynamic system) and the largest eigenvalue of $H$, which describes the "energy" in the system. Recall that a unique positive definite $H$ is found through solution of the Lyapunov equation Eq. (4) for our stable subsystems ($A_i$'s).

By using the characteristic #1 for testing $M$-matrix conditions we can see that $W \in M$ if

$$a^2 - 2a\zeta > 0 \tag{8}$$

$$a - \zeta > 0 \tag{9}$$

where it is seen that Eq. (8) describes the determinant of the matrix $W$ and Eq. (9) is the first principal minor of $W$, equivalent to the second principal minor of this simple example matrix. Depending upon the resulting value of the parameter $a$ for this system we are able to place bounds on the connective stability of this dynamic system by aggregating the energy and dissipation of the subsystems and explicitly accounting for interconnection parameters. This technique is powerful in that we are able to greatly reduce the composite problem dimensionality and examine stability as a function of subsystem interconnectivity strength.

Importantly, we now relate this back to the nuclear-physical system of interest: by modeling the salt vault interconnections in a way where they explicitly show up as parts of the $\zeta_{ij}$ terms in the $W$ matrix for the SmAHTR system we will be able to see how close the composite system gets to the connective stability boundary by varying the strength of these interconnection terms (which will be a direct function of the mass of salt, $m_s$, in the salt vault) in combination with subsystem stability properties. That is, the scalar Lyapunov functions are used to aggregate the relative stability of subsystems in their respective portions of the state space.

This technique will allow for individual, independent, SmAHTR reactor modeling and control design to be done. If a controller gain scheduling approach is taken for individual reactor subsystems or for the composite system Ref. [13, 16], Lyapunov functions can be found for these "scheduled", controlled systems and immediately fit into vector Lyapunov theory for testing the connective stability of the composite, controlled system.

# 5 RESULTS AND ANALYSIS

## 5.1 Stability of the Composite System as a Function of Salt Vault Size

Without the use of vector Lyapunov theory we can evaluate the stability of the composite system for the two boundary cases discussed in this paper: 1.) for a salt vault of infinite (very large) size;

2.) for a salt vault if infinitesimal (very small) size. Of course, for a physically-realizable system we cannot entertain the notion of "infinite salt vault size" or the situation where the salt vault is non-existent. We therefore assign some numerical values for each situation. For the case where we encourage strong coupling between reactor systems and Brayton cycles we assume that the salt vault has an effective mass of $10^2$ kg. For the case where we encourage weak, or the decoupling, of reactor systems and Brayton cycles we assume that the salt vault has an effective mass of $10^{10}$ kg. These results are given in table 1. Clearly, the composite system is stable in both cases since

**Table 1: Min and max eigenvalues of composite system as function of salt vault mass**

| $m_s$(kg) | $\lambda_m$ | $\lambda_M$ |
|---|---|---|
| $10^2$ | $-0.012$ | $-4.802 \times 10^2$ |
| $10^{10}$ | $-5.054 \times 10^{-7}$ | $-4.802 \times 10^2$ |

all $\mathrm{Re}[\lambda(\cdot)] < 0$ for these salt vault sizes. This implies that the composite, decentrally-controlled reactor systems can be stable without a salt vault present. We also see that when the mass of the salt vault is made very large, the salt vault will dominate composite system dynamics. This is evidenced by the purely negative real eigenvalue very close to the origin. This is an obvious observation since we see that by manipulating the mass of the salt vault, we are varying a time constant (and hence eigenvalue) in the salt vault. Our previous discussion about the salt vault serving as a very large "low pass filter" between plant subsystems is supported by this observation.

Qualitatively-speaking, it may have been predicted that the composite system would remain stable for both of these cases. This analysis quantitatively verifies this condition, and we point out a key shortcoming to this approach: we have little basis for judging how the composite system's ability to dissipate energy under strong interconnections is affected. This is a feature that vector Lyapunov theory affords us, along with the ability to interchange portions of the composite system state space dynamics for testing the connective stability of multiple controlled reactor configurations.

### 5.2 Stability of the Composite System as a Function of Salt Vault Size — A Vector Lyapunov Approach

We have discussed large scale system theory and the use of vector Lyapunov functions for testing stability in terms of parameterized interconnections in the multi-module SMR control problem. Parameterization of salt vault interconnection dynamics with reactor subsystems and Brayton cycles is an ongoing activity, however, we can illustrate, roughly, what the structure of the $W$ matrix for a 2-reactor / salt vault plant would look like. This structure is shown in figure 5. The red boxes around the diagonal elements indicate the terms in this matrix that would change if subsystem controllers or dynamics were adjusted. That is, if we were to elect to change the way that the core controller was designed for reactor #2, the closed-loop dynamics of reactor #2 would change. In this situation, the $w_{33}$ element would be affected by such a change in plant subsystem dynamics, whereas the $w_{11}$ element (representing salt vault stability) and $w_{22}$ element (representing reactor #1 stability) would go unchanged. This "plug-and-test" approach makes testing of numerous plant configurations for stability and "closeness" to a stability boundary attractive. Interconnection terms, as shown previously, manifest in both the off-diagonal and diagonal elements of the $W$ matrix ($\zeta_{ij}$ terms). As subsystem interconnection strength increases, we see that we can get closer to $W \notin M$. This illustrates how we get closer to a stability boundary and how well the composite plant will be able to withstand energy/power transients.

## 6 CONCLUSIONS

This paper reports the development of a large scale dynamic model of the SmAHTR reactor Ref. [5]. Reactor subsystems are controlled using traditional PI and linear quadratic integrator approaches.

$$W_e = \begin{bmatrix} \boxed{\frac{1}{2}\frac{\lambda_m(G_1)}{\lambda_M(H_1)}} - \zeta_{11} & -\zeta_{12} & -\zeta_{13} \\ -\zeta_{21} & \boxed{\frac{1}{2}\frac{\lambda_m(G_2)}{\lambda_M(H_2)}} - \zeta_{22} & 0 \\ -\zeta_{31} & 0 & \boxed{\frac{1}{2}\frac{\lambda_m(G_3)}{\lambda_M(H_3)}} - \zeta_{33} \end{bmatrix}$$

**Figure 5: Principal $W$-matrix terms affected by different reactor configurations/operating regimes, different reactor controller designs. This leaves the composite system largely unchanged and allows for checking stability when portions of the state space are perturbed.**

All four reactors are coupled to a salt vault and three Brayton cycles and the composite system stability is analyzed as a function of salt vault size using simple techniques from linear system theory.

We then introduce some powerful techniques from the field of large scale system theory that use vector Lyapunov functions for testing the connective stability of large scale systems with emphasis on subsystem interconnections. Using these techniques, we show how stability testing of very high dimensional models of dynamic systems can be reduced to a problem that is of dimension equal to the number of subsystems in a given dynamic system. Judicious choice and appropriate modeling of interconnection parameters in the form of Eq. (3) is a necessary component of this work and leads to the eventual parameterization of interconnection parameters for testing connective stability.

Using large scale system theory to analyze the stability of multi-unit SMRs offers several advantages:

1. Reduction of dimensions and the parameterization of key plant interconnections for stability analysis.

2. Allows for system designers and analysts to test the connective stability of multiple reactor configurations, controller designs, and operating points without disturbing other regions of the state space. This enables somewhat of a "plug and test" approach to evaluating connective stability.

3. Provides an entry point for multi-module SMR control problems into the rich field of large scale system theory.

## 7 FUTURE WORK

Current efforts are directed toward casting the SmAHTR plant into the large scale system form described in this paper. This activity will appropriately extract composite plant interconnection terms to form the $A_{ij}$ matrices that connect subsystems through the salt vault. Using static values for the salt vault mass, connective stability will be verified using the techniques described herein for testing for $M$-matrix conditions. Thereafter, the mass of the salt vault will be parameterized and bounds on connective stability will be found. It was shown in this work for these given plant configurations/controlled reactor designs that the composite system will be connectively stable. The use of vector Lyapunov theory can lead to conservative bounds on stability due to the nature by which the positive definite matrix $G$ is chosen in solving the Lyapunov equation Eq. (4). Much work has gone into formulating system transformations that reduce this conservatism, with current results as reported by Ref. [6] and associated authors leading to optimal choices for the matrix $G$ used in Eq. (4). If deemed necessary and appropriate, these transformations will be investigated for use with this SmAHTR model. Ultimately, an optimization problem can be pursued that merges these results with some measure, or cost, that brings salt vault economics into the equation. This would provide an interesting way of balancing stability, performance, and economics through the use of powerful tools in control theory and optimization methods.

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