REDUCED ORDER METHODS FOR THE IMPROVEMENT OF CONTROL-ORIENTED MODELLING OF NUCLEAR POWER PLANTS

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ABSTRACT

In the development of the control system of a nuclear power plant, it is of primary importance to rely on simulation tools for its realization, testing and validation. These tools should provide an accurate description of the controlled response of the entire power plant in different scenarios. The classic control-oriented modelling based on 0D/1D approach is well suited for control purposes since it provides fast-running simulations and it is easy to linearize, but it may not be sufficient to deeply investigate the complexity of some systems, in particular when/where spatial phenomena have a remarkable impact on dynamics. Reduced Order Methods (ROMs) can offer the proper trade-off between computational cost and solution accuracy, combining the high-detail modelling (e.g., 3D modelling) usually adopted for design purposes with the requirements demanded for a control-oriented tool, firstly the computational efficiency. In this work, ROMs are used to improve a control-oriented plant simulator of a Lead-cooled Fast Reactor (LFR). In particular, the attention is focused on the reactor core neutronics, and on the thermal-hydraulics of the reactor pool as well. A test case regarding the design of a Kalman Filter is set up showing promising results for the application of ROM in the control context.

Key Words: Reduced Order Methods, Kalman Filter, Control, Modelling

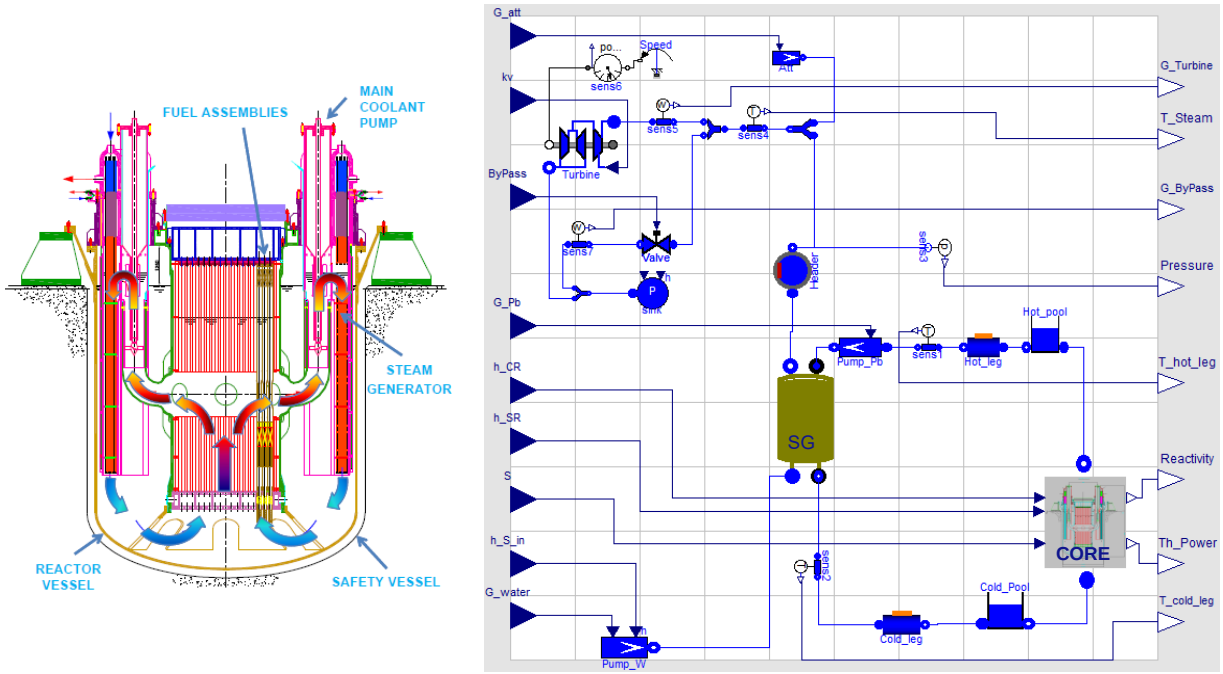
1 INTRODUCTION

In the development of the control system, it is of primary importance to rely on simulation tools for its realization, testing and validation by means of an accurate description of the reactor-controlled response. The classic control-oriented approach based on 0D/1D modelling is appropriate whether the spatial effects are not relevant and only the estimation of integral quantities is required as for the Single Input Single Output (SISO) control laws. On the opposite side, the 3D modelling is usually devoted to design purposes having a high level of detail but extremely expensive from a computational point of view. A viable solution for combining a high-detail modelling featuring spatial capabilities (e.g., 3D modelling) with the requirements demanded for a control-oriented tool, firstly the computational efficiency, is represented by Reduced Order Methods (ROMs) [1-3]. The main assumption of ROMs is that the behaviour of the system with respect to a parameter (physical, geometrical) or the time can be represented by a small number of dominant modes. In this way, the system evolution is represented by a reduced set of Ordinary Differential Equations (ODEs). The latter can be employed, for instance, as the basis for the synthesis and the verification of controllers [4].

The subject of this work is the use of reduced order methods for the improvement of control-oriented modelling of nuclear power plants. The high accuracy guaranteed by the adoption of ROM-based models,

along with the ODEs representation, allows solving some control issues related to modelling aspects, in particular the spatial ones, which otherwise could not be managed by means of the classic control-oriented approach. The reference reactor considered in this paper is ALFRED (Advanced Lead Fast Reactor European Demonstrator) [5], a small-size (300 MWth) pool-type reactor (Fig. 1a) whose conceptual design has been developed within the Euratom LEADER Project, even if the proposed approach and modelling techniques can be applied to any reactor concept. In this work, the modelling effort is focused on the neutronics and the reactor pool components of a power plant simulator (Fig. 1b). As possible application of the ROM approach in the control context, a Kalman Filter of a test case is designed starting from the ROM-based model developed for the reactor pool.

The paper is organized as follows. In Section 2, a general overview on the ROMs is presented. In Sections 3 and 4, the description of the ROM-based modelling for neutronics and thermal-hydraulics are provided. In Section 5, the Kalman Filter designed from the ROM-based model of the reactor pool is described along with some numerical data. Finally, some conclusions, perspectives and further developments are given in Section 6.



(a) (b)

Figure 1. (a) ALFRED nuclear power plant layout [5], (b) ALFRED plant simulator [6].

2 REDUCED ORDER METHODS

Reduced order methods is a generic expression to identify any approach aimed at replacing a high-fidelity problem, i.e., the Full Order Model (FOM), by one featuring a much lower computational complexity, i.e., the reduced order model [7]. The purpose of a computational reduction technique is to retain the governing dynamics of a system in a rapid and reliable way. In particular, the reduced order modelling is aimed at approximating a Partial Differential Equation (PDE) solution (or a set of ODEs) with a reduced number of degrees of freedom. The common procedure is to solve the full-order problem only for a properly selected number of instances of the input parameter (through a demanding Offline computational step), in order to be able to perform many low-cost real-time simulations (inexpensive Online computational step) for new instances of the parameter [2]. The essential components of the procedure can be summarized as follows [2]:

- a high-fidelity discretization technique aimed at calculating some high-fidelity PDE solutions (called snapshots), which are needed to build the reduced order basis.
- a (Galerkin) projection. The reduced order solution is usually expressed as a linear combination of the basis functions. The coefficients of this combination are calculated by means of Galerkin-like projection of the equations onto the reduced order space.
- an offline/online procedure. The expensive computation of the snapshot and the basis calculation can be performed just once (offline phase) and totally decoupled from the fast-running simulation of the reduced order model.
- an error estimation procedure both to assess the accuracy of the ROM-based model and to construct a reliable, certified, and suited reduced order basis (the development of an error estimation procedure is out of the scope of this work).

The selection of the spatial basis and test functions involved in the projection step is a crucial issue in the development of the ROM model. In particular, the aim is selecting the optimal pair of spatial basis/test functions that maximizes the accuracy of the model and/or minimizes the computational cost. A classic option is the Modal Method (MM) that employs as spatial basis the eigenfunctions of the governing equations. The Modal Method provides a "general" spatial basis since it is related to the eigenvalue equation of the system. An alternative option can be constituted by the adoption of the POD with the snapshot technique [8,9], which is aimed at using low dimensional approximations of a high dimensional system according to a "maximum energy/info" criterion. The Proper Orthogonal Decomposition is able to provide "ad hoc" spatial basis tailored on the specific simulation case, thanks to its optimality properties.

3 ROM-BASED MODELLING FOR NEUTRONICS

A ROM-based spatial model for the reactor core neutronics has been developed in order to go beyond the classic Point Kinetics (PK) currently used in control-oriented models. The main shortcoming of such zero-dimensional method is the inability to allow for the spatial dependence of the flux. This prevents the use of advanced model-based control schemes, for instance, to optimize the Control Rod (CR) handling. In addition, the PK does not allow considering the different contributes of each zone to the reactivity thermal feedback. For additional information, the reader may refer to [10,11].

3.1 Modelling approach

For the description of the neutron kinetics, the multi-group diffusion theory [12], with a generic number G of energy groups and eight groups of precursors, is considered. In equations (matrix formulation), it reads:

$$\underline{\underline{V}}^{-1}\frac{\partial \underline{\phi}}{\partial t} = \nabla\cdot\left(\underline{\underline{D}}\nabla\underline{\phi}\right) - \underline{\underline{\Sigma_a}}\,\underline{\phi} - \underline{\underline{\Sigma_s}}\,\underline{\phi} + (1-\beta)\underline{\chi_p}\underline{F}^T\underline{\phi} + \sum_j \lambda_j \underline{\chi_d} C_j \tag{1}$$

$$\frac{\partial C_j}{\partial t} = -\lambda_j C_j + \beta_j \underline{F}^T\underline{\phi} \qquad j = 1 \div 8 \tag{2}$$

where $\underline{\underline{V}}$ is the matrix of the neutron speed, $\underline{\phi}$ is the vector of the neutron flux, $\underline{\underline{D}}$ is the matrix of the neutron diffusion coefficient, $\underline{\underline{\Sigma_a}}$ is the matrix of the absorption cross-section, $\underline{\underline{\Sigma_s}}$ is the matrix of the scattering cross-section, β is the total delayed neutron fraction, $\underline{\chi_p}$ is the vector of the fraction of prompt neutrons, $\underline{F}$ is the vector of the fission cross section, λ_j is the decay constant of the j[th] precursor group, $\underline{\chi_d}$ is the vector of the fraction of delayed neutrons, C_j is the concentration of the j[th] precursor group, β_j is the delayed neutron fraction of the j[th] precursor group. The neutron flux can be expressed as

$$\underline{\phi}(\boldsymbol{r},t) \cong \sum_{i=1}^{N} \underline{\underline{\psi_i}}(\boldsymbol{r})\cdot\underline{n_i}(t) \tag{3}$$

where $\underline{\psi}_i(r)$ is the matrix of the spatial basis where the flux is projected, $\underline{n}_i(t)$ is the vector of the time-dependent coefficients and N is the number of employed functions in the spatial basis. In order to transform the multi-group diffusion PDEs into a set of ODEs involving only the time-dependent coefficient $\underline{n}_i(t)$, the expression of Eq. (3) has to be substituted into Eqs. (1) and (2), the latter have to be multiplied by test functions $\underline{\xi}_i$ and integrated over the computational domain. This procedure can be related to a Petrov-Galerkin projection. Finally, the ODE system for the time-dependent coefficients can be expressed, for each basis function, as

$$\sum_{m=1}^{N} \underline{\underline{RV}}_{im} \cdot \dot{\underline{n}}_m = \sum_{m=1}^{N} \left(-\underline{\underline{L}}_{im} - \underline{\underline{\delta L}}_{im} + (1-\beta) \cdot \left(\underline{\underline{M}}_{im} + \underline{\underline{\delta M}}_{im} \right) \right) \cdot \underline{n}_m + \sum_{j=1}^{8} \lambda_j \underline{c}_{ij} \tag{4}$$

$$\dot{\underline{c}}_{ij} = \beta_j \underline{\underline{X}} \left[\sum_{m=1}^{N} \left(\underline{\underline{M}}_{im} + \underline{\underline{\delta M}}_{im} \right) \cdot \underline{n}_m \right] - \lambda_j \underline{c}_{ij} \quad j = 1 \div 8 \tag{5}$$

where

$$\begin{gathered} \underline{\underline{RV}}_{im} = \int \underline{\xi}_i \cdot \underline{\underline{V}}^{-1} \cdot \underline{\psi}_m d\Omega, \quad \underline{\underline{L}}_{im} = \int \underline{\xi}_i \cdot \left(-\nabla \cdot \underline{\underline{D}} \nabla + \underline{\underline{\Sigma}}_a + \underline{\underline{\Sigma}}_s \right) \underline{\psi}_m d\Omega, \quad \underline{\underline{M}}_{im} = \int \underline{\xi}_i \cdot \left(\underline{\chi}_p \underline{F}^T \right) \underline{\psi}_m d\Omega, \\ \underline{\underline{\delta L}}_{im} = \int \underline{\xi}_i \cdot \delta \left(-\nabla \cdot \underline{\underline{D}} \nabla + \underline{\underline{\Sigma}}_a + \underline{\underline{\Sigma}}_s \right) \underline{\psi}_m d\Omega, \quad \underline{\underline{\delta M}}_{im} = \int \underline{\xi}_i \cdot \delta \left(\underline{\chi}_p \underline{F}^T \right) \underline{\psi}_m d\Omega, \\ \underline{c}_{ij} = \int \underline{\xi}_i \cdot \underline{\chi}_d \cdot C_j \, d\Omega \qquad \underline{\underline{X}} = \begin{bmatrix} \chi_d^1/\chi_p^1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \chi_d^G/\chi_p^G \end{bmatrix} \end{gathered} \tag{6}$$

L_{im} and M_{im} represent the contribution to the removal and production operator calculated in the unperturbed system. These quantities are calculated once in the "offline" process, and are kept constant during the transient simulation. δL_{im} and δM_{im} represent the variation of the removal and the production operators during the transients. On one hand, the variation can be due to the temperature change of the cross-sections, i.e., the reactivity thermal feedbacks. On the other hand, the removal and production operator can change due to the CR movement. For the neutronics calculation, the geometry is divided in coarse zones. For each region, the reactivity insertion is weighted on the spatial basis functions and the test functions integrated over the zone, considering the operator variation constant. In this way, the calculation of the integral between the spatial basis and the test functions over the zone can be performed once during the offline process, and it is kept constant during the transient simulation.

Two methods for the spatial basis selection are undertaken, namely the Modal Method and the Proper Orthogonal Decomposition. Another degree of freedom is the choice of the test functions $\underline{\xi}_i$. Typically, they are the same functions that constitute the spatial basis as in a Galerkin projection. Nevertheless, the test functions can be different as in a Petrov-Galerkin projection. In this sense, a possible option is to employ the adjoint functions related to the spatial basis due to the property of bi-orthogonality. This choice assumes a particular meaning in the neutronics field since it is related to the neutron importance. The latter option (called "Adjoint Proper Orthogonal Decomposition") has been proven to give better results [11].

4 ROM-BASED MODELLING FOR THERMAL-HYDRAULICS

As for the thermal-hydraulics, a POD-Galerkin Method for Finite Volume Approximation of Navier-Stokes and RANS equations (POD-FV-ROM) has been developed [13]. This approach is directed to overcome the 0D/1D modelling usually employed in control-oriented models for the fluid dynamics. In particular, the 0D/1D approach prevents the simulation tool from taking into account the spatial features of the fluid flow, which can be relevant for certain reactor systems. The aim of this new ROM approach is both to extend the classic POD-Galerkin-ROM (POD-G-ROM) method [14] to the Finite Volume approximation of the Navier-Stokes equations and to build a reduced order model that is capable to handle turbulent flows modelled through the RANS equations.

4.1 Modelling approach

In the following, the POD-FV-ROM technique for RANS equations with eddy viscosity turbulent modelling is summarized. For additional information, the reader may refer to [11,13]. If we consider a general incompressible RANS eddy viscosity model, the equations of the FOM read:

$$\begin{cases} \boldsymbol{u}_t + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} = \nabla\cdot[-p\boldsymbol{I} + (\upsilon+\upsilon_t)(\nabla\boldsymbol{u} + (\nabla\boldsymbol{u})^T) - 2/3k\boldsymbol{I}] \\ \nabla\cdot\boldsymbol{u} = 0 \end{cases} \tag{7}$$

where $\mathbf{u}$ is the velocity, p is a normalized pressure, υ is the kinematic viscosity, and υ_t the turbulent viscosity. The equations are given in a domain Ω with proper boundary conditions (BCs) and initial conditions. The turbulent viscosity υ_t is usually function of one ($\tilde{\upsilon}$ in Spalart-Allmaras) or two variables (k and ε/ω in the respective models).

The main assumption in the ROMs based on projection technique is that the approximated solution of the problem $\boldsymbol{u}_r(\boldsymbol{x},t)$ can be expressed as linear combination of spatial modes $\boldsymbol{\varphi}_i(\boldsymbol{x})$ multiplied by temporal coefficients $a_i(t)$. If we consider the velocity, this assumption reads

$$\boldsymbol{u}(\boldsymbol{x},t) \approx \boldsymbol{u}_r(\boldsymbol{x},t) = \sum_{i=1}^{N} a_i(t)\boldsymbol{\varphi}_i(\boldsymbol{x}) \tag{8}$$

The POD basis $\boldsymbol{\varphi}_i$ can be built starting from a set of N_s velocity solutions $\boldsymbol{u}_n(\boldsymbol{x})$ sampled at different and evenly spaced times (i.e., the snapshots). The POD basis minimizes the difference between the snapshots and the projection of the snapshots on the spatial modes. In order to calculate the POD basis, the following eigenvalue problem is considered:

$$C\xi_i = \lambda_i\xi_i \qquad i = 1,\dots,N_s \qquad [C]_{kl} = \frac{1}{N}\langle \boldsymbol{u}_k(\boldsymbol{x}), \boldsymbol{u}_l(\boldsymbol{x})\rangle_{L^2} \tag{9}$$

where $C \in \mathbb{R}^{N_s \times N_s}$ is the Correlation Matrix. The (λ_i, ξ_i) eigenvalue – eigenvector pair is used to construct the functions of the POD basis

$$\boldsymbol{\varphi}_i(\boldsymbol{x}) = \frac{1}{\sqrt{\lambda_i}}\sum_{n=1}^{N_s} \xi_{i,n}\boldsymbol{u}_n(\boldsymbol{x}) \qquad i = 1,\dots,N_r \tag{10}$$

The procedure described so far is close to the classic methodology [14]. Hereinafter, the method moves away from the classic one due to our aim to adopt the ROM approach in a FV framework, and to consider the turbulence based on RANS equations with eddy viscosity turbulent modelling. To this aim, the face flux F, the pressure and the turbulent viscosity can be expressed as linear combination of some spatial modes:

$$F(\boldsymbol{x},t) \approx \sum_{i=1}^{N} a_i(t)\psi_i(\boldsymbol{x}), \qquad p(\boldsymbol{x},t) \approx \sum_{i=1}^{N} a_i(t)\chi_i(\boldsymbol{x}), \qquad \upsilon_t(\boldsymbol{x},t) \approx \sum_{i=1}^{N} a_i(t)\phi_i(\boldsymbol{x}) \tag{11}$$

where $\psi_i(\boldsymbol{x})$, $\chi_i(\boldsymbol{x})$ and $\phi_i(\boldsymbol{x})$ are the functions of the POD spatial basis for the face flux, the pressure and the turbulent viscosity, respectively. These spatial bases are constructed considering the eigenvectors of the correlation matrix of the velocity (Eq. 10) and the snapshots of the face flux, pressure and turbulent viscosity ($F_n(\boldsymbol{x})$, $p_n(\boldsymbol{x})$ and $\upsilon_{t,n}(\boldsymbol{x})$, obtained from the full order model) as follows:

$$\psi_i(\boldsymbol{x}) = \frac{1}{\sqrt{\lambda_i}}\sum_{n=1}^{N_s} \xi_{i,n}F_n(\boldsymbol{x}) \quad \chi_i(\boldsymbol{x}) = \frac{1}{\sqrt{\lambda_i}}\sum_{n=1}^{N_s} \xi_{i,n}p_n(\boldsymbol{x}) \quad \phi_i(\boldsymbol{x}) = \frac{1}{\sqrt{\lambda_i}}\sum_{n=1}^{N_s} \xi_{i,n}\upsilon_{t,n}(\boldsymbol{x}) \quad i = 1,\dots,N_r, \tag{12}$$

Replacing the velocity $\boldsymbol{u}$ with $\boldsymbol{u}_r$, p with p_r and υ_t with $\upsilon_{t,r}$ in the Eq. (8), employing the approximated face flux F_r in the convective term, applying the Galerkin projection, and employing a penalty factor τ [15] in order to enforce/parametrize the BCs ($\mathbf{u}_{BC}$), the following POD-Galerkin ROM for Finite Volume discretization (POD-FV-ROM) is obtained:

$$\frac{da_i(t)}{dt} = \upsilon \sum_{i=1}^{N} B_{ji}\, a_i(t) + \upsilon \sum_{i=1}^{N} BT_{ji}\, a_i(t) - \sum_{k=1}^{N}\sum_{i=1}^{N} C_{jki}\, a_k(t) a_i(t) + \sum_{k=1}^{N}\sum_{i=1}^{N} CT1_{jki}\, a_k(t) a_i(t) + \sum_{k=1}^{N}\sum_{i=1}^{N} CT2_{jki}\, a_k(t) a_i(t) - \sum_{i=1}^{N} A_{ji}\, a_i(t) + \tau\left(\boldsymbol{u_{BC}} \cdot \boldsymbol{D}_j - \sum_{i=1}^{N} E_{ji}\, a_i(t)\right) \tag{13}$$

where

$$a_j(0) = \langle \boldsymbol{\varphi}_j, \boldsymbol{u}_1(\boldsymbol{x}) \rangle_{L^2}, A_{ji} = \langle \boldsymbol{\varphi}_j, \nabla \chi_i \rangle_{L^2}, B_{ji} = \langle \boldsymbol{\varphi}_j, \Delta \boldsymbol{\varphi}_i \rangle_{L^2}, BT_{ji} = \langle \boldsymbol{\varphi}_j, \nabla \cdot (\nabla \boldsymbol{\varphi}_i^{\boldsymbol{T}}) \rangle_{L^2}, C_{jki} = \langle \boldsymbol{\varphi}_j, \nabla \cdot (\psi_k, \boldsymbol{\varphi}_i) \rangle_{L^2},$$
$$CT1_{jki} = \langle \boldsymbol{\varphi}_j, \phi_k \Delta \boldsymbol{\varphi}_i \rangle_{L^2}, CT2_{jki} = \langle \boldsymbol{\varphi}_j, \nabla \cdot \phi_k (\nabla \boldsymbol{\varphi}_i^{\boldsymbol{T}}) \rangle_{L^2}, \boldsymbol{D}_j = \langle \boldsymbol{\varphi}_j \rangle_{L^2,\partial\Omega}, E_{ji} = \langle \boldsymbol{\varphi}_j, \boldsymbol{\varphi}_i \rangle_{L^2,\partial\Omega} \tag{14}$$

The dynamical system of the time-dependent coefficients for turbulence can be expressed in the following matrix form:

$$\dot{\boldsymbol{a}} = \upsilon(\boldsymbol{B} + \boldsymbol{BT})\boldsymbol{a} - \boldsymbol{a}^{\boldsymbol{T}}(\boldsymbol{C} - \boldsymbol{CT1} - \boldsymbol{CT2})\boldsymbol{a} - \boldsymbol{Aa} + \tau(\boldsymbol{u_{BC}D} - \boldsymbol{Ea}) \tag{15}$$

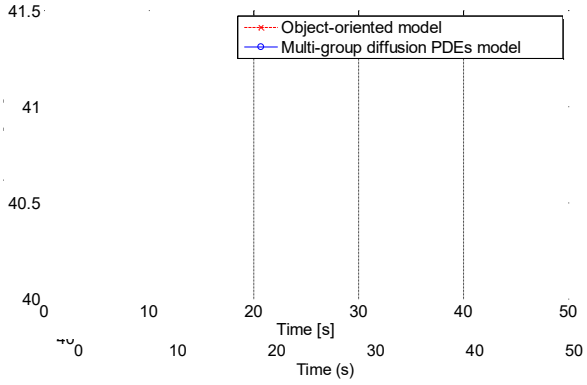
5 NUMERICAL RESULTS

The ROM-based modelling for neutronics and thermal-hydraulics has been applied for the improvement of a control-oriented simulator of a Lead-cooled Fast Reactor [6], i.e., substituting some components based on zero-dimensional approach with ROM-based models. The simulator (Fig. 1b) is based on the object-oriented modelling, is developed with Modelica language [16] and implemented in the Dymola simulation environment [17]. In the ROM framework, Modelica with its component approach turns out to be a powerful tool since it is possible to update or substitute a component with the respective ROM-based one without compromising the rest of the model.

As for the spatial neutronics modelling, an object-oriented model of a 3D test case of the ALFRED reactor has been settled in order to prove the feasibility of employing ROM-based components in control-oriented simulators (Fig. 2a). As for the thermal-hydraulics, a parametric ROM-based component of the coolant pool of the ALFRED reactor has been developed [18] (Fig. 3a).

```
Model Spatial Neutronics
parameter Real A1NP[N_eigen*KP1.N_groups,N_eigen*KP1.N_groups]=[-
1.385680e+008,⋯,-3.248002e+006];
⋯
Real C[N_eigen*KP1.N_groups*KP1.NPG,1];
Real npsi_n[N_eigen*KP1.N_groups,1];
⋯
Interfaces.Reactivity React_Pin_1;
equation
der(n_i.n) = ((A1NP-A1NP_rho*rho_in)/(1-rho_in))*n_i.n+
React_Pin_1.Pert_L+ React_Pin_2.Pert_L+ React_Pin_3.Pert_L +
A2NP*C;
der(C) = (A3NP/(1-rho_in))*n_i.n+ React_Pin_1.Pert_M+
React_Pin_2.Pert_M+ React_Pin_3.Pert_M+A4NP*C;
⋯
End Spatial Neutronics;
```

(a)



(b)

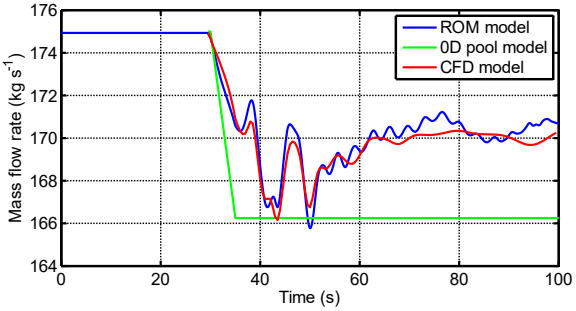
Figure 2. (a) Modelica code for the neutronics component; (b) Pin power following a lead inlet temperature enhancement. Comparison between multi-group diffusion (reference) and object-oriented model.

To demonstrate the possibility of employing a ROM-based component in a control-oriented simulator, the possibility to vary the input variables of the model has been undertaken. In particular, the lead velocity at the Steam Generator (SG) outlet has been considered as parametrized boundary condition since it can be a possible control variable. Both the ROM-based models turn out to be able to reproduce the transient behaviour of the main variables of interest with a degree of accuracy close to a high-fidelity (full order) model - multi-group diffusion PDEs model and CFD model, see Fig. 2b and 3b - but with a reduced

computational time (45 s against 300 cpu-h of the high-fidelity model in a 50s transient for neutronics, and 25 s against 212 cpu-h for a single simulation of the high fidelity model in a 70s transient for thermal-hydraulics).

```
model Coolant_Pool_ROM  "ROM model for ALFRED coolant pool";
   parameter Integer N=30 "Number of functions in the basis";
   parameter Real A[:,:]=readMatrix("matrices.mat", "A", N,N)
   ...
   Real a[N,1]; "time coefficients"
equation   "Eq. 31"
  der(a)=nu*(B+BT)*a+C_function(a)-
A*a+tau(inlet_SG_1.m_flow*DSG1+inlet_SG_2.m_flow*DSG2-E*a);
  FA_int_1.m_flow=-FA_1_flow[1,:]*a[:,1];
initial equation
   a=transpose(ci);
end Coolant_Pool_ROM;
```

(a)



(b)

Figure 3. (a) Modelica code for the coolant pool component; (b) Evolution of the mass flow rate in the central fuel assembly following an asymmetric velocity variation of the lead velocity at SG outlet of 10% with respect to the nominal value. ROM (blue line), 0D (green line) and CFD (red line) results.

5.1 ROM – Kalman filter

In order to prove the potentialities of the ROM approach in the control context, a possible application has been set up involving the Kalman Filter (KF). The KF can be seen as a state observer aimed at estimating the internal state ($\hat{x}_k$) of a system considering inputs (u_k) and some measurements of the outputs of the real system (z_k), which can be affected by noise (Fig. 4). In particular, the KF is divided in two steps [19]. The first one, the *predictor* step, is meant to give an a-priori estimate of the state ($\hat{x}_k^-$) and the output ($\hat{z}_k^-$) starting from a process model, along with the projection of the error covariance (P_k^-). In equations, it reads:

$$\hat{x}_k^- = A_{KF}\hat{x}_{k-1} + B_{KF}u_k \quad (16)$$

$$\hat{z}_k^- = H_{KF}\hat{x}_k^- \quad (17)$$

$$P_k^- = A_{KF}P_{k-1}A_{KF}^T + Q \quad (18)$$

where the matrix A_{KF} relates the state at the prevision time step to the state ($\hat{x}_{k-1}$) at the current time step ($\hat{x}_k^-$), the matrix B_{KF} relates the input (u_k) to the state ($\hat{x}_k^-$), the matrix H_{KF} relates the state ($\hat{x}_k^-$) to the estimate outputs ($\hat{z}_k^-$) that will be compared with the real measurements (z_k), and Q is the process noise covariance matrix. The second step, the *corrector* step, tries to obtain an a-posteriori estimate of the state ($\hat{x}_k$) starting from the calculation of the Kalman gain (K_k) and considering the measurements (z_k), along with the update of the error covariance (P_k^-). In equations, it reads:

$$K_k = P_k^- H^T (HP_k^- H^T + R)^{-1} \quad (19)$$

$$\hat{x}_k = \hat{x}_k^- + K_k(z_k - \hat{z}_k^-) \quad (20)$$

$$P_k = (I - K_k H)P_k^- \quad (21)$$

where R is the measurement noise covariance matrix. It is clear that in order to design a KF, it is relevant to build a model of the system to be employed in the predictor step. Usually the model employed is a first principles model, i.e., 0D/1D modelling or a state-space representation. In addition, the measurements that are used for the corrector step are usually integral variables that can be also predicted by the model. On the other hand, if we want to improve the estimation capabilities of the KF, using also the measurements that come from specific points of our system, we need a model that takes into account the spatial feature of the system. In this case, the direct use of PDE equations is not possible due to the high number of the states and the need of a proper post-processing. To this aim, the ROM approach could be useful since *i)* the ROM-based model is characterized by a low number of states (i.e., the time coefficients of the spatial basis) and it is in an ODE form which allows an easily linearization; *ii*) we can relate the spatial information measurement with the state of the system since we can easily reconstruct the field of interest by means of Eq. (8) and/or Eq. (11).

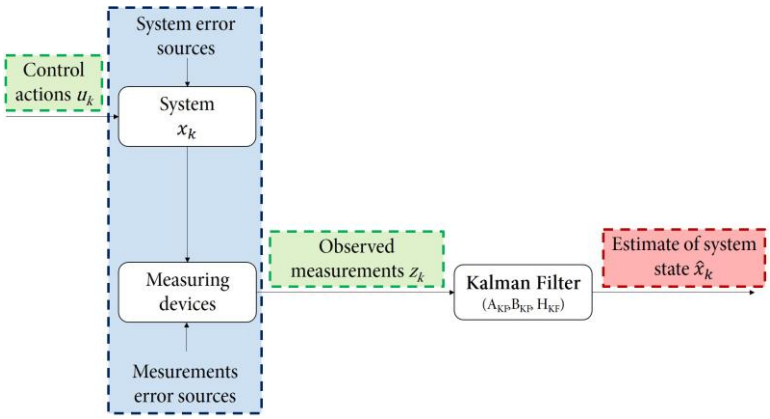
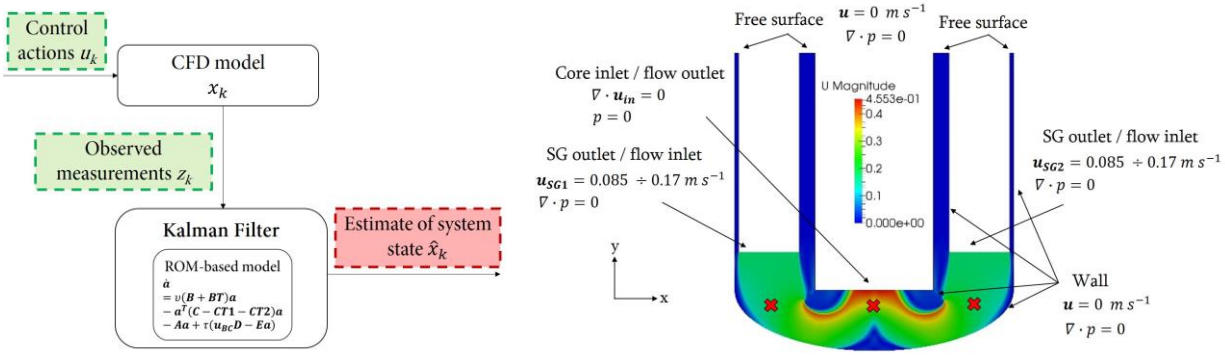


Figure 4. Kalman filter concept (adapted from [20]).

A test case, involving the coolant pool of the ALFRED as system characterized by spatial features, has been set up in order to prove the feasibility of employing ROM-based models in control context. In particular, the design of a KF (ROM-KF) for the estimation of the velocity field inside the coolant pool of ALFRED has been performed, starting from the modelling approach presented in Section 4 (Fig. 5a). As for the corrector step, three velocity measurements, generated by a 2D CFD simulation of the coolant pool, are considered (Fig. 5b). Following the description of the modelling given in Section 4, the matrices for the KF based on ROM model are built as follows:

$$\boldsymbol{A}_{KF}(t) = \upsilon(\boldsymbol{B} + \boldsymbol{BT}) - \boldsymbol{a}^{\boldsymbol{T}}(\boldsymbol{C} - \boldsymbol{CT1} - \boldsymbol{CT2}) - \boldsymbol{A} - \tau\boldsymbol{E} \tag{22}$$

$$\boldsymbol{B}_{KF}(t) = \tau\boldsymbol{D};\ \boldsymbol{H}_{KF} = \boldsymbol{H}_{ROM},\quad H_{ROMj,i} = \varphi_i(\boldsymbol{x_j});\ \hat{x} = \boldsymbol{a};\ u = \boldsymbol{u_{BC}} \tag{23}$$



(a) (b)

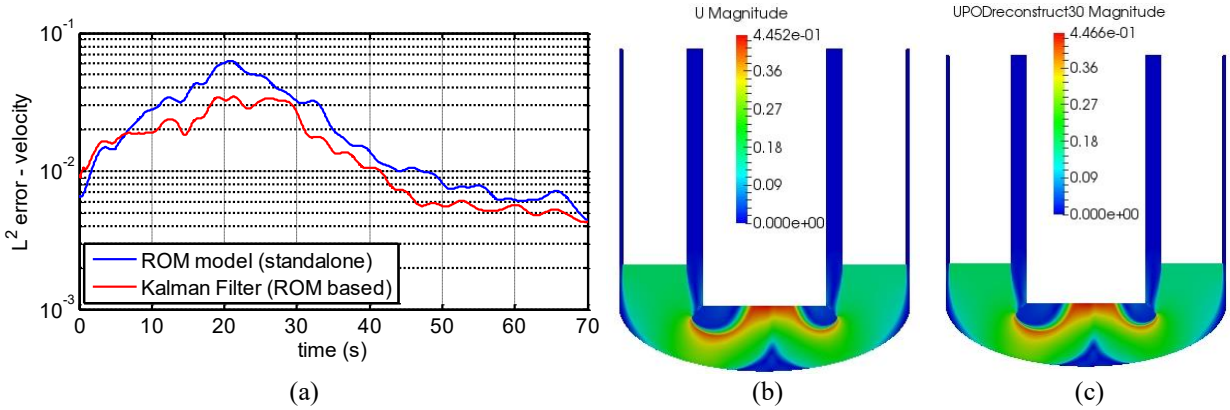
Figure 5. (a) Kalman filter concept used for the test case; (b) 2D geometry and boundary conditions of the ALFRED pool model. In the background, contours of velocity magnitude (m/s) in nominal condition. The location of the three fictitious measurements are marked with the red cross.

To assess the accuracy of the KF estimate, the difference between the velocity reconstructed from the estimated state of the KF and FOM model (taken as reference) is reported using the L^2 error:

$$\|e\|_{L^2} = \sqrt{\frac{\langle(\boldsymbol{u_{FOM}} - \boldsymbol{u_r}), (\boldsymbol{u_{FOM}} - \boldsymbol{u_r})\rangle_{L^2}}{\langle \boldsymbol{u_{FOM}}, \boldsymbol{u_{FOM}}\rangle_{L^2}}} \tag{24}$$

In Fig. 6a, the L^2 error of the reconstructed velocity of a standalone ROM model and the ROM-KF are shown with 30 basis functions following a velocity variation of the second SG of 10% with respect to the nominal value. Both the standalone and the ROM-KF models are able to reconstruct the velocity field being the L^2 error lower than $5 \cdot 10^{-2}$. In particular, thanks to the integration with the spatial measurements, the

ROM-KF shows improved results being the error usually lower than the ROM standalone one. The contours of the velocity magnitude at the end of the transient (t=70 s) for the reference FOM model and for the ROM-KF are show in Figs. 6b and 6c, respectively. The simulation time is comparable with the use in a real-time application (tens of seconds for a simulated time of 70 s).



(a) (b) (c)

Figure 6. (a) L^2 error of the reconstructed velocity from standalone ROM model (blue line) and ROM-KF (red line) with N=30 following a velocity variation of 10% - u_{SG2}=0.153 m/s with respect to the nominal value; contours of velocity magnitude (m/s) at the end of the transient for the FOM model (b) and the ROM-KF (c).

6 CONCLUSIONS

In this work, the control-oriented modelling of nuclear reactors is improved through the development of reduced order methods. This new methodology in the control-oriented modelling of the nuclear reactor is meant to overcome the classic approach based on 0D/1D modelling. The latter is in general appropriate for estimating integral quantities but it is of poor detail when the spatial dependence plays a relevant role in the respect of technological constraints or in general in the dynamics evolution. This work is aimed at combining the high-detail modelling usually adopted for design purposes (e.g., 3D modelling) with the requirements demanded for a control-oriented tool, firstly the computational efficiency. The high accuracy guaranteed by the adoption of ROM-based models allows solving some control issues related to modelling aspects, in particular the spatial ones, which otherwise could not be managed by means of the classic control-oriented approach. As for the neutronics, a ROM-based spatial model for the reactor core has been developed aimed at improving the classic PK approach used in control-oriented modelling. As for the thermal-hydraulics, a parametric ROM-based component of the coolant pool of the ALFRED reactor has been developed directed to overcome the 0D/1D modelling usually employed in control-oriented models for the fluid dynamics. The results show that the behaviour of the ROM-based component is more accurate than 0D model without an excessive computational cost. As a major outcome, a possible application of the ROM approach in the control context has been tested. In particular, the design of a Kalman Filter for the estimation of the velocity field inside the coolant pool of ALFRED has been performed, starting from the ROM-based model developed for the thermal-hydraulics. The results show that the ROM-KF is able to correctly estimate the variables of interest (in this case the velocity) taking advantage of the information coming from the measurements. Moreover, thanks to the data assimilation, the results are improved with respect to a standalone ROM-based model. In conclusion, the ROM approach can be of value in order to both improve the control-oriented simulation and to be employed in control context (e.g., in the synthesis of controllers). The main advantages in this sense are the low number of states, the accurate representation of the dynamics (compared to the 0D/1D modelling) with a reduced computational time (compared to the 3D modelling) and the ability to provide information of the spatial feature of the system. Several further developments can be pursued in the future, starting from a systematic study of the “control” properties of the ROM-based model as controllability, observability and stability, along with the development of advanced control schemes (e.g., optimal control) based on the ROM approach.

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